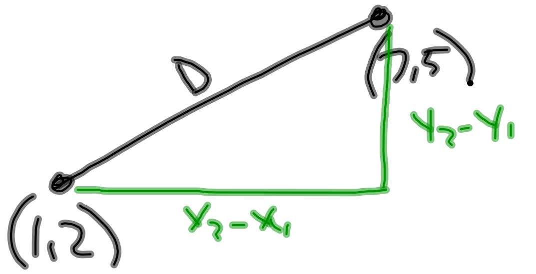
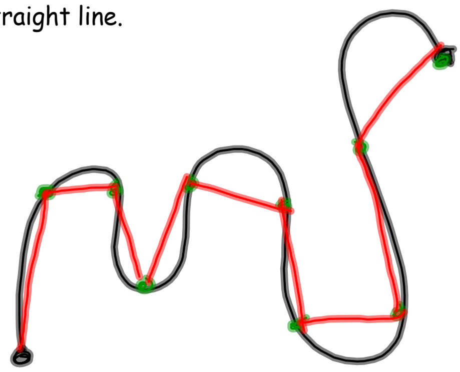
Math 152 Calculus and Analytic Geometry II

Sec. 8.1 Arc Length

How do you calculate the length of a curve. Divide it into many segments and approximate each one by a straight line. Use the distance formula on each straight line.



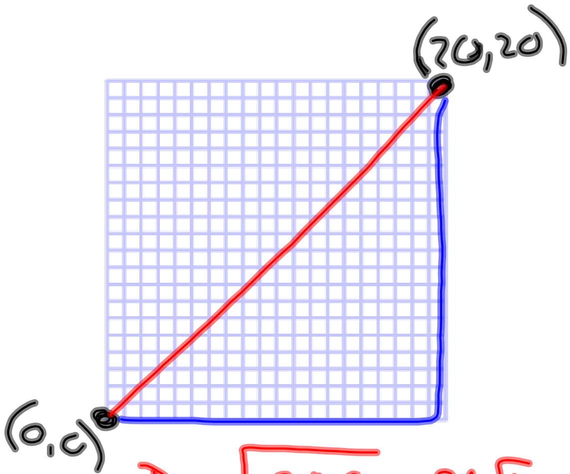
$$D=\sqrt{(7-1)^2+(5-2)^2}$$

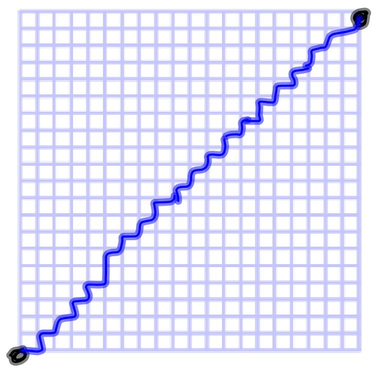


Red line segments approximate length of curve.

We can derive a nice integral formula for this as long as the function has a continuous derivative.

If a function does not have a continuous derivative, we can't use this formula





$D=\sqrt{800}=20\sqrt{2}$

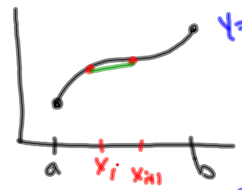
Blue Distance = 40

Blue length is "still" 40

http://www.shodor.org/interactivate/activities/kochsnowflake/

Suppose we have a curve y=f(x) on an interval from a to b.

Divide it into segments, how long is the segment with width dx?



$y=f(x)$ from a to b

length of green line segment

$$D=\sqrt{(x_{i+1}-x_i)^2+(f(x_{i+1})-f(x_i))^2}$$

$$D=\sqrt{(\Delta x)^2+(\Delta y)^2}$$

$$D=\sqrt{(\Delta x)^2+\left(\frac{\Delta y}{\Delta x}\right)^2(\Delta x)^2}$$

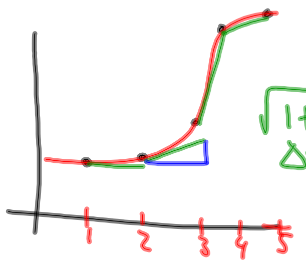
$$=\Delta x\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}$$

So what is the integral for the total length?

$\frac{\text{rise}}{\text{run}}$ = derivative $= f'(x)$ in the limit

$$L=\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\cdot\Delta x$$

$$L=\int_a^b\sqrt{1+(f'(x))^2}\,dx$$



$\sqrt{1+(f')^2}=$

$\Delta x=1$

Find the length of the curve $y=\frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}}$ between x=1 and x=3

Draw it and estimate the length before you start.

$$L=\int_1^3\sqrt{1+(f'(x))^2}\,dx$$

$$f'(x)=1(x^2-1)^{1/2}\cdot(2x)$$

$$(f'(x))^2=(x^2-1)(4x^2)$$

$$=4x^4-4x^2$$

$$=\int_1^3\sqrt{4x^4-4x^2+1}\,dx$$

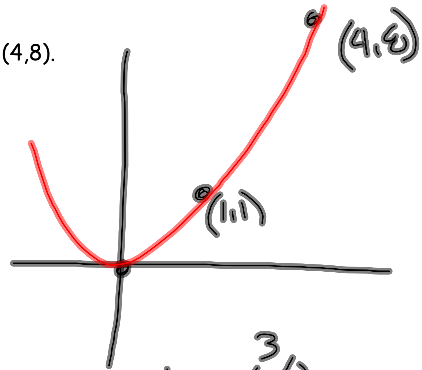
$$=\int_1^3\sqrt{(2x^2-1)^2}\,dx$$

$$=\int_1^3 2x^2-1\,dx$$

$$=\left.\frac{2x^3}{3}-x\right]_1^3=\left(\frac{54}{3}-3\right)-\left(\frac{2}{3}-1\right)$$

$$=46/3\ ?$$

Find the length of the curve $y^2 = x^3$ from (1,1) to (4,8).



$y = x^{3/2}$

$f'(x) = \frac{3}{2}x^{1/2}$

$(f'(x))^2 = \frac{9}{4}x$

$$L = \int_1^4 \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_1^4 \sqrt{1+\frac{9}{4}x}\,dx$$

substitute $u = 1+\frac{9}{4}x$

$du = \frac{9}{4}dx$

$\frac{4}{9}du = dx$

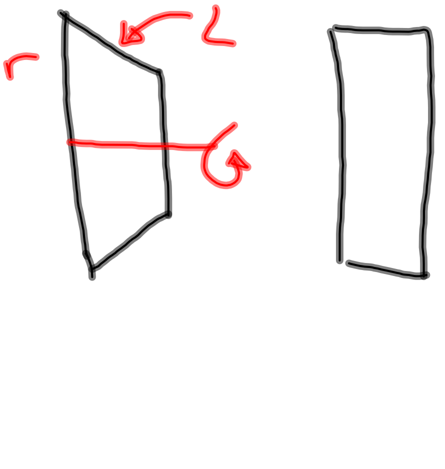
$$\int_{13/4}^{10} \sqrt{u}\cdot\frac{4}{9}du = \frac{4}{9}\cdot\frac{2}{3}u^{3/2}\Big]_{13/4}^{10} = ?$$

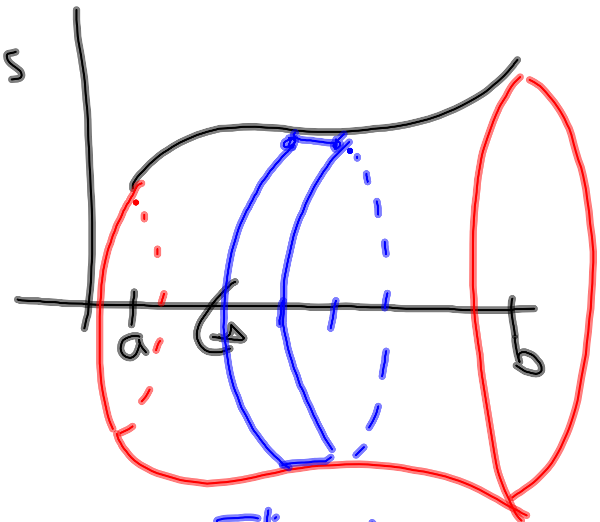
Sec. 8.2 Surface Area (Quickly)

rotate around x-axis

$$SA=\int_a^b 2\pi\underbrace{(f(x))}_{\text{radius}}\underbrace{\sqrt{1+\left(f'(x)\right)^2}}\,dx$$

$$SA = 2\pi(radius)(diagonal - lentgh)$$

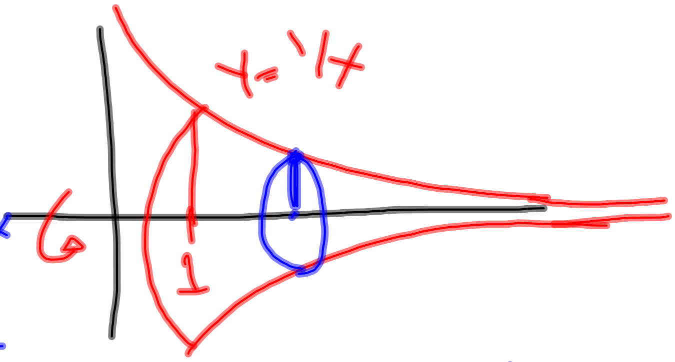




Slice is a cylinder

Rotate y=1/x around the x-axis from x=1 to infinity.

Find Volume.



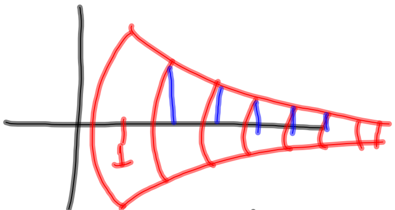
$$V=\lim_{t\to\infty}\int_1^t \pi\left(\frac{1}{x}\right)^2 dx$$

$$=\lim_{t\to\infty} \pi\left(-x^{-1}\right)\Big]_1^t$$

$$=\lim_{t\to\infty} -\frac{\pi}{t}-\left(-\frac{\pi}{1}\right) = 0+\frac{\pi}{1} = \pi \text{ unit}^3$$

http://www.calculusapplets.com/revolution.html

Find Surface Area.



$$SA=\lim_{t\to\infty}\int_1^t 2\pi\underbrace{\left(\frac{1}{x}\right)}_{\text{radius}}\sqrt{1+\left(\frac{1}{x^4}\right)}\,dx$$

$y=1/x$

$f'(x)=-x^{-2}$

$(f'(x))^2=x^{-4}$

We can't integrate this, but

$$=\lim_{t\to\infty}\int_1^t 2\pi\frac{1}{x}\left(\sqrt{1+x^{-4}}\right)dx$$

$$\geq \lim_{t\to\infty}\int_1^t 2\pi\frac{1}{x}\,dx$$

$$=\lim_{t\to\infty} 2\pi\ln|x|\Big|_1^t$$

$$=\lim_{t\to\infty} 2\pi\ln|t|-2\pi\ln(1)$$

$$=\infty-2\pi(0)$$

$$=\infty$$

Midterm III

$$\lim_{x\to 0} (\sin(x))^x \qquad 0^0\text{- form}$$

indeterminate

$$\lim_{x\to 0} e^{\ln((\sin(x))^x)}$$

$$\lim_{x\to 0} e^{x\ln(\sin(x))}$$

Find $\lim_{x\to 0} x\cdot \ln(\sin(x))$

$$\lim_{x\to 0} \frac{\ln(\sin x)}{1/x} = \lim_{x\to 0} \frac{\frac{1}{\sin(x)}\cdot \cos(x)}{-1/x^2}$$

LH's rule

$$= \lim_{x\to 0} -x\cdot \frac{x}{\sin(x)}\cdot \cos(x)$$

$$= (0)(1)(1)$$

$$= 0$$

so $e^0 = \boxed{1}$

$1^\infty \qquad 0^0$

$1\cdot 1\cdot 1\cdot 1\cdot 1 \ldots\ldots$

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$$

$$(1.000000001)^{1000000000}$$

$= e$

Compounded 12 times a year

$$A = P\cdot\left(1+\frac{r}{12}\right)^{12t}$$

$$e^{rt}$$