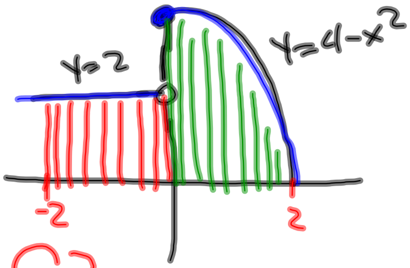
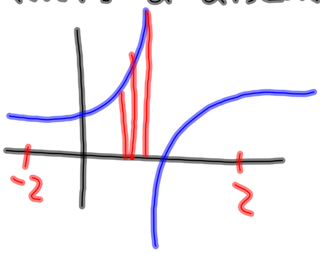
Questions from HW 5.3

(8) $\int_{-2}^{2} f(x)\,dx$



$$= \int_{-2}^{0} 2\,dx + \int_{0}^{2} 4-x^2\,dx$$

When is a discontinuity a problem



cant break it into 2 intervals that we can evaluate.

$$\int_0^1 10^x\,dx$$

Not $\frac{10^{x+1}}{x+1}$

$$\frac{1}{\ln(10)} \cdot 10^x \Big]_0^1 = \frac{10}{\ln(10)} - \frac{1}{\ln(10)}$$

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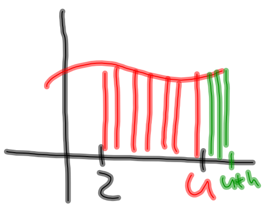
(4) $h(u) = \int_2^u \sqrt{1+r^3}\,dr$

$h'(x) = \sqrt{1+(u)^3}$



new Area $= h \cdot (\sqrt{1+u^3})$

$h(x) = \int_2^{x^2} \sqrt{1+r^3}\,dr$

$h'(x) = \sqrt{1+(x^2)^3} \cdot 2x$

inside

derivative of inside

2) Consider the function $-\sqrt{2-x^2}$. Use geometry to compute the value of $\int_0^{\sqrt{2}} -\sqrt{2-x^2}\,dx$.

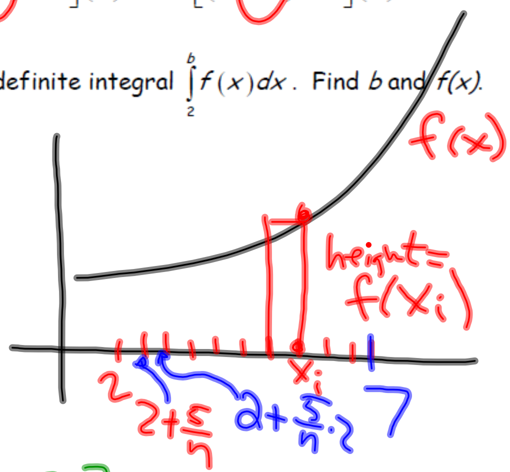
3)

a) Express the following sum in the form $\sum_{i=1}^{n} a_i$.

$$\left[\left(2+\frac{5}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\left[\left(2+\frac{10}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\left[\left(2+\frac{15}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\cdots+\left[\left(2+\frac{5n}{n}\right)^3+1\right]\left(\frac{5}{n}\right)$$

b) The above is a right Riemann Sum for the definite integral $\int_2^b f(x)\,dx$. Find b and $f(x)$.

$$\sum_{i=1}^{n}\left[\left(\underbrace{2+\frac{5i}{n}}_{x_i}\right)^3+1\right]\cdot\left(\frac{5}{n}\right)$$



$x_i = 2+\frac{5i}{n}$

$\Delta x = \frac{b-a}{n} = \frac{5}{n}$

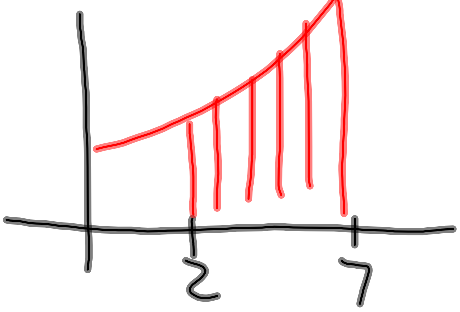
$f(x) = x^3+1$

$= \int_2^7 (x^3+1)\,dx$

c) Find the value of the above sum as $n \to \infty$.

d) The above sum can also be a right Riemann Sum for the definite integral $\int_0^b f(x)dx$. Find the new function and bounds and compute the value of the sum as $n \to \infty$.

$$\int_2^7 (x^3+1)\,dx$$



3)

a) Express the following sum in the form $\sum_{i=1}^{n} a_i$.

$$\left[\left(2+\frac{5}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\left[\left(2+\frac{10}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\left[\left(2+\frac{15}{n}\right)^3+1\right]\left(\frac{5}{n}\right)+\cdots+\left[\left(2+\frac{5n}{n}\right)^3+1\right]\left(\frac{5}{n}\right)$$

$x_i = \frac{5}{n} i$

b) The above is a right Riemann Sum for the definite integral $\int_2^b f(x)dx$. Find b and $f(x)$.

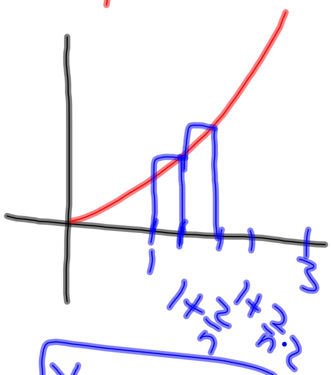
$$\int_0^5 \left((2+u)^3+1\right)du$$

$u+2=x$

Substitute

4) Express the definite integral $\int_1^3 x^2\,dx$ as a right Riemann Sum and compute the sum using sigma algebra and the limit as $n \to \infty$. Verify that the sum you compute is the same as computing the integral using the Fundamental Theorem of Calculus.

$$\int_1^3 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\overbrace{\left(\left(1+\frac{2}{n}i\right)^2\right)}^{\text{height}}}_{f(x_i)} \cdot \underbrace{\overbrace{\left(\frac{2}{n}\right)}^{\Delta x}}_{\left(\frac{b-a}{n}\right)}$$



$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2}{n}i\right)^2\left(\frac{2}{n}\right)$$

$$\boxed{x_i = 1+\frac{2}{n}i}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2}{n}i\right)^2\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{4}{n}i+\frac{4}{n^2}i^2\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} + \sum_{i=1}^{n} \frac{8}{n^2}i + \sum_{i=1}^{n} \frac{8}{n^3}i^2$$

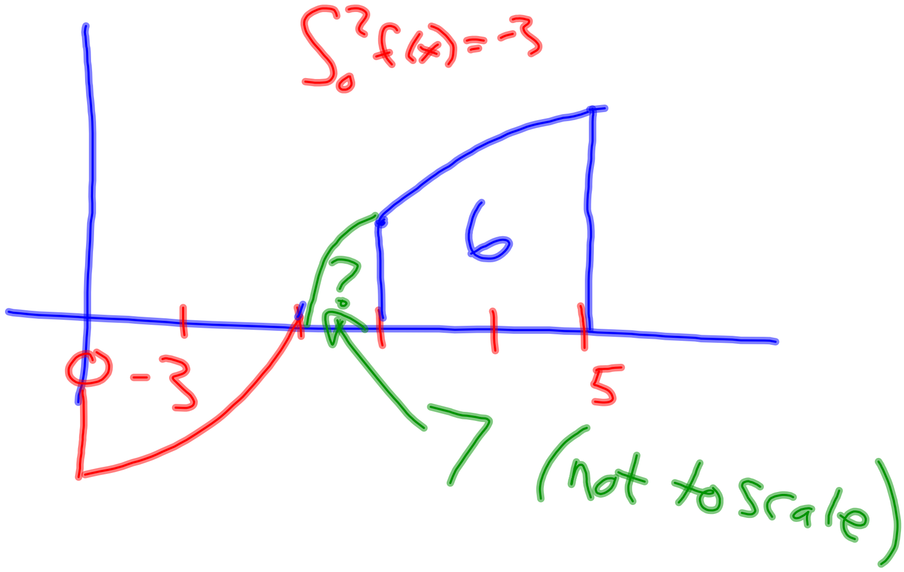
$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} 1 + \frac{8}{n^2}\sum_{i=1}^{n} i + \frac{8}{n^3}\sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{2}{n}(n) + \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 2 + 4 + 8\cdot\frac{2}{6}$$

5) Suppose $\int_0^5 f(x) = 10, \int_2^0 f(x) = 3, \int_3^5 f(x) = 6$. Then $\int_2^3 f(x) = ?$

$\int_0^2 f(x) = -3$



6) Find the following integrals:

$u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}dx$

a. $\int \frac{\sec^2 x}{\tan^6 x} dx$ b. $\int (\cos(2\theta) - 2\sin\theta)\, d\theta$ c. $\boxed{\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx}$

d. $\boxed{\int (x+1)e^{(x+1)^2} dx}$ e. $\int \frac{2}{x(\ln x^2)^2} dx$ f. $\int_{-1}^{2} (3x^2 + x)dx$

g. $\int_1^5 \sqrt{2x-1}\,dx$ h. $\int_0^1 x^3(4-5x^4)^6 dx$ i. $\int_{-2}^{2} |x-1|\,dx$

$u = x+1$

$du = dx$

$\cancel{\int u e^{u^2} du}$

Harder

$u = (x+1)^2$

$du = 2(x+1)\cdot(1)dx$

$\frac{1}{2}\int e^u du$

7) a. Let $g(x)=\int_1^x \cot^5\left(\sqrt{t}\right)dt$. Find $g'(x)$.

$g'(x)=\cot^5\left(\sqrt{x}\right)$

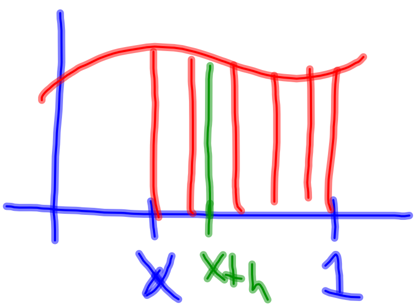
b. Let $h(x)=\int_{x^3}^1 \cot^5\left(\sqrt{t}\right)dt$. Find $h'(x)$. (note that $1 \le x$ must be true)

or Flip it

$h'(x)=$

$h=-\int_1^{x^3}\cot^5\left(\sqrt{t}\right)dt$

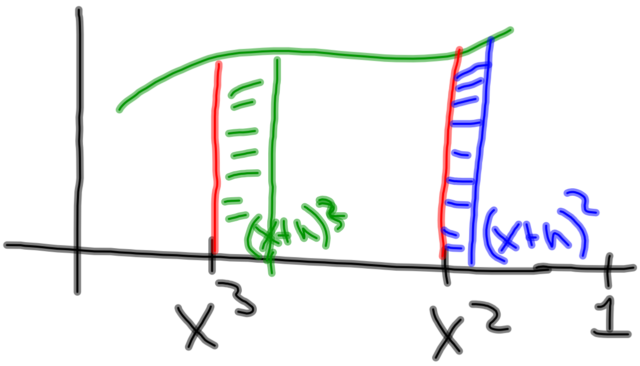
$h'(x)=-\cot^5\left(\sqrt{x^3}\right)\cdot 3x^2$



when x increases
Area is decreasing
So derivative is negative.

c. Let $f(x)=\int_{x^3}^{1}\frac{d}{dt}\cot^5\left(\sqrt{t}\right)dt$. Find $f'(x)$. (note that $1\le x$ must be true)

d. Let $f(x)=\int_{x^3}^{x^2} e^{t^2}\sin(2t)\,dt$. Find $f'(x)$.



chain rule

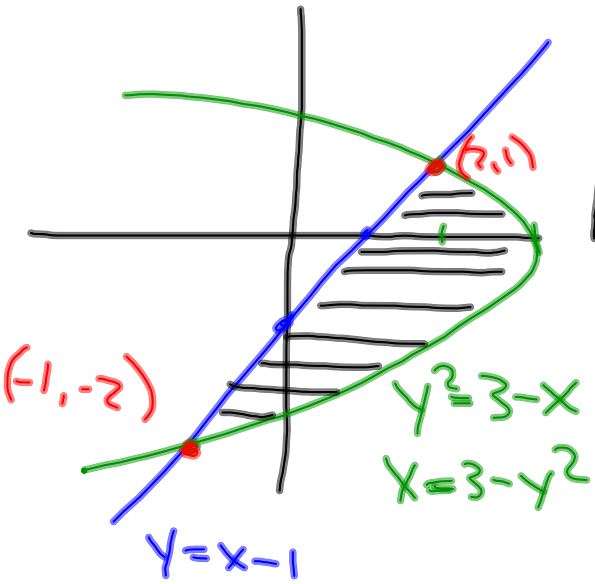
$$f'(x)=e^{(x^2)^2}\sin(2(x^2))(2x)$$ added area

$$-e^{(x^3)^2}\sin(2(x^3))\,3x^2$$ subtracted area

9) a. Find the area between the graphs of $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$.

b. Find the area between the graphs of $y = x - 1$ and $y^2 = 3 - x$.

$x = y + 1$



$$\text{Area} = \int_{-2}^{1} (3-y^2)-(y+1)\,dy$$

TRUE-FALSE QUIZ

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

True integrate term by term

2. If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x)g(x)]\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$$

False products are more complicated

3. If f is continuous on $[a, b]$, then

True $$\int_a^b 5f(x)\,dx = 5\int_a^b f(x)\,dx$$ factor out constant

4. If f is continuous on $[a, b]$, then

False $$\int_a^b xf(x)\,dx = x\int_a^b f(x)\,dx$$ Cant factor out variable

5. If f is continuous on $[a, b]$ and $f(x) \geqslant 0$, then

$$\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$$

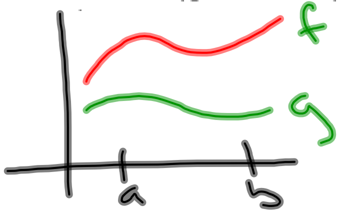
False

6. If f' is continuous on $[1, 3]$, then $\int_1^3 f'(v)\,dv = f(3) - f(1)$.

True F.T.C. part II

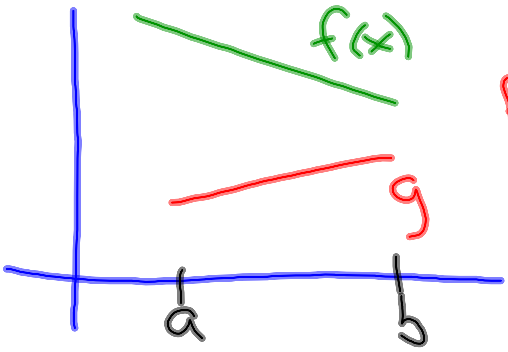
7. If f and g are continuous and $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then

$$\int_a^b f(x)\,dx \geqslant \int_a^b g(x)\,dx$$



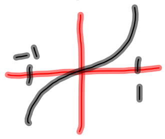
True

8. If f and g are differentiable and $f(x) \geqslant g(x)$ for $a < x < b$, then $f'(x) \geqslant g'(x)$ for $a < x < b$.

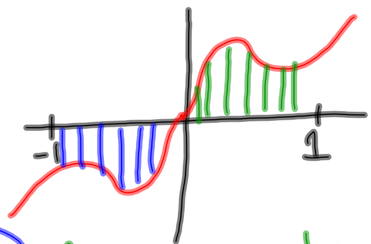


False

9. $\int_{-1}^{1} \left(x^5 - 6x^9 + \frac{\sin x}{(1 + x^4)^2} \right) dx = 0$

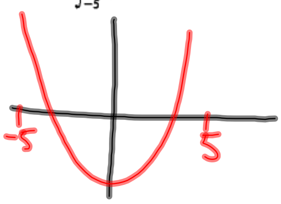


odd functn



Areas cancel in odd functns.

10. $\int_{-5}^{5} (ax^2 + bx + c)\,dx = 2\int_0^5 (ax^2 + c)\,dx$



False

Even functns you can integrate one half and double it.

11. $\int_{-2}^{1} \frac{1}{x^4}\,dx = -\frac{3}{8}$

$\cancel{\int x^{-4}\,dx}$ $\quad \left.\frac{x^{-3}}{-3}\right]_{-2}^{1} = \frac{1}{-3} - \frac{(-2)^{-3}}{-3} = \frac{-3}{8}$

Asymptote in interval

13. All continuous functions have derivatives.

14. All continuous functions have antiderivatives.

15. If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x)$$

9–38 Evaluate the integral, if it exists.

9. $\int_1^2 (8x^3 + 3x^2)\,dx$

10. $\int_0^T (x^4 - 8x + 7)\,dx$

11. $\int_0^1 (1 - x^9)\,dx$

12. $\int_0^1 (1 - x)^9\,dx$

13. $\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

14. $\int_0^1 (\sqrt[4]{u} + 1)^2\,du$

15. $\int_0^1 y(y^2 + 1)^5\,dy$

16. $\int_0^2 y^2\sqrt{1 + y^3}\,dy$

17. $\int_1^5 \frac{dt}{(t - 4)^2}$

18. $\int_0^1 \sin(3\pi t)\,dt$

43–48 Find the derivative of the function.

43. $F(x) = \int_0^x \frac{t^2}{1 + t^3}\, dt$

44. $F(x) = \int_x^1 \sqrt{t + \sin t}\, dt$

45. $g(x) = \int_0^{x^4} \cos(t^2)\, dt$

46. $g(x) = \int_1^{\sin x} \frac{1 - t^2}{1 + t^4}\, dt$

47. $y = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\, dt$

48. $y = \int_{2x}^{3x+1} \sin(t^4)\, dt$

56. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 5]$.

57. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000, and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t)\,dt$ represent?

what does $f(x) = \int_0^x r(t)dt$ mean?

Find $f'(x)$

67. If f' is continuous on $[a, b]$, show that

$$2\int_a^b f(x)f'(x)\,dx = [f(b)]^2 - [f(a)]^2$$

$u = f(x)$
$du = f'(x)dx$

$$2\int_a^b u\,du$$

$$2\left(\frac{u^2}{2}\right)\Big]_a^b = \frac{\cancel{2}}{1}\frac{f(x)^2}{\cancel{2}}\Big]_a^b = [f(b)]^2 - [f(a)]^2$$

68. Find $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + t^3}\, dt$.

69. If f is continuous on $[0, 1]$, prove that

$$\int_0^1 f(x)\, dx = \int_0^1 f(1 - x)\, dx$$

70. Evaluate

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right]$$

PROBLEMS PLUS

2. Find the minimum value of the area of the region under the curve $y = x + 1/x$ from $x = a$ to $x = a + 1.5$, for all $a > 0$.

4. (a) Graph several members of the family of functions $f(x) = (2cx - x^2)/c^3$ for $c > 0$ and look at the regions enclosed by these curves and the x-axis. Make a conjecture about how the areas of these regions are related.
(b) Prove your conjecture in part (a).
(c) Take another look at the graphs in part (a) and use them to sketch the curve traced out by the vertices (highest points) of the family of functions. Can you guess what kind of curve this is?
(d) Find an equation of the curve you sketched in part (c).

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6. If $f(x) = \int_0^x x^2 \sin(t^2)\, dt$, find $f'(x)$.

9. Find the interval $[a, b]$ for which the value of the integral $\int_a^b (2 + x - x^2)\, dx$ is a maximum.

12. Find $\dfrac{d^2}{dx^2} \displaystyle\int_0^x \left(\int_1^{\sin t} \sqrt{1 + u^4}\, du \right) dt.$

Attachments

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