Midterm 2
Math 152
Spring 2010
May 10, 2011
Dr. Maharry

Name SOLUTIONS

Justify your answers by showing all of your work for each problem. Partial credit will be given based on the work shown that is leading to a correct answer. Correct answers with no supporting work will NOT receive full credit. Please Circle your final answers. Good Luck.

(100 points)

1. (6 points each) Evaluate the following integrals by substitution:

$\int \sqrt{Sin(2x)}Cos(2x)dx$

$u = \sin(2x)$

$du = \cos(2x) \cdot 2 \cdot dx$

$\frac{du}{\cos(2x) \cdot 2} = dx$

$= \int \sqrt{u} \, \frac{\cancel{\cos(2x)}\, du}{\cancel{\cos(2x)} \cdot 2}$

$= \frac{1}{2}\int \sqrt{u}\, du$

$= \frac{1}{2}\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(\sin(2x))^{3/2} + C$

$\int \frac{xSec^2\left(\sqrt{x^2+4}\right)}{\sqrt{x^2+4}}dx$

$u = x^2+4$

$du = 2x\,dx$

$= \frac{1}{2}\int \frac{\sec^2(\sqrt{u})}{\sqrt{u}}du = \int \sec^2(v)dv$

$v = \sqrt{u}$

$dv = \frac{1}{2\sqrt{u}}du$

$= \tan(v) + C$

$= \tan\left(\sqrt{x^2+4}\right) + C$

$$-\frac{1}{3}\int_0^2 \frac{-3x^2}{(9-x^3)^{\frac{3}{2}}}\,dx \quad = \quad -\frac{1}{3}\int_9^1 \frac{1}{u^{3/2}}\,du$$

$u = 9 - x^3$

$du = -3x^2\,dx$

$$= \frac{1}{3}\int_1^9 u^{-3/2}\,du$$

$$= \frac{1}{3}\cdot\frac{-2}{1}\cdot u^{-1/2}\Big]_1^9$$

$$= -\frac{2}{3}\left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{1}}\right) = -\frac{2}{3}\left(\frac{1}{3} - 1\right)$$

$$= \frac{4}{9}$$

$$\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}\,dx$$

$u = 3/x$

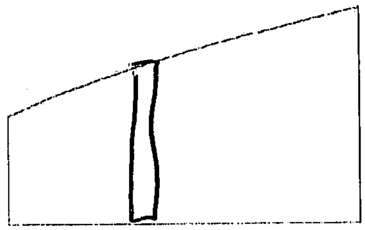
$du = -3/x^2\,dx$

$$-\frac{1}{3}\int_3^1 e^u\,du = -\frac{1}{3}\,e^u\Big]_3^1$$

$$= -\frac{1}{3}(e - e^3)$$

$$= \frac{e^3 - e}{3}$$

2. (8 points each part) Consider the region drawn below. It is bounded by $y=\sqrt{x}$ $\quad y=0 \quad x=1 \quad x=4$. Draw the axes and label the corners of the region.



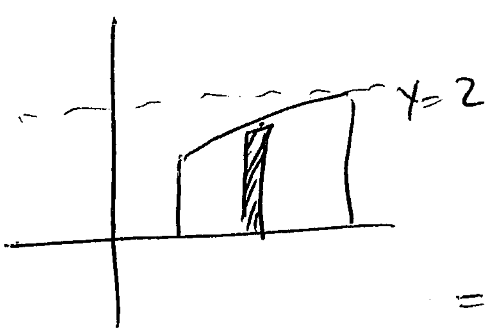
a. Use "vertical rectangles" to set up an integral to find the Area of the Region. Find the Area.

$$A = \int_1^4 (\sqrt{x} - 0)\,dx$$

$$= \frac{2}{3}x^{3/2}\Big]_1^4 = \frac{2}{3}(8) - \frac{2}{3}(1)$$

$$= \frac{14}{3}$$

b. Use "vertical rectangles" to set up an integral to find the Volume of the Solid obtained by rotating the region around the line y=2. Find the Volume.



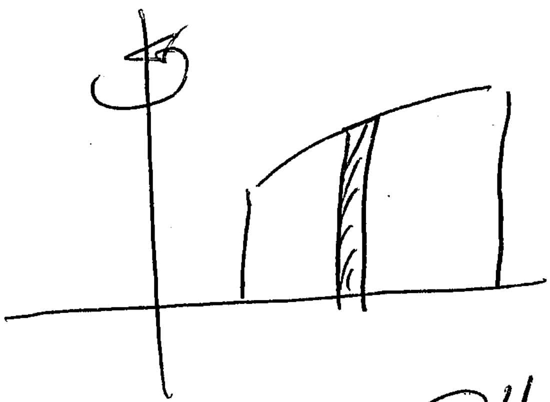
$$V = \int_1^4 \pi(2-0)^2 - \pi(2-\sqrt{x})^2\,dx$$

$$= \int_1^4 4\pi - (4\pi - 4\pi\sqrt{x} + \pi x)\,dx$$

$$= \cancel{4\pi x} - \cancel{4\pi x} + 4\pi \frac{2}{3} x^{\frac{3}{2}} - \frac{\pi x^2}{2}\Big]_1^4$$

$$= \left(\frac{8\pi}{3}(8) - \frac{\pi(16)}{2}\right) - \left[\frac{8\pi}{3}(1) - \frac{\pi(1)}{2}\right]$$

c. Use "vertical rectangles" to set up an integral to find the Volume of the Solid obtained by rotating the region around the y-axis. Find the Volume.



$$V = \int_1^4 2\pi(x-0)(\sqrt{x}-0)\,dx$$

$$= \int_1^4 2\pi\, x\sqrt{x}\,dx$$

$$= \int_1^4 2\pi\, x^{3/2}\,dx$$

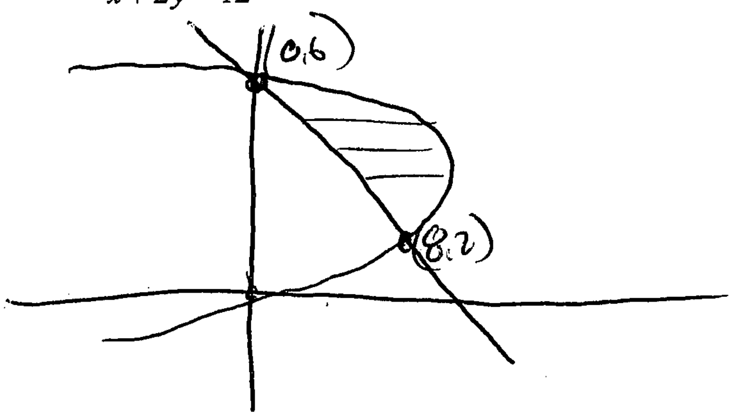
$$= 2\pi\, \frac{2}{5} x^{5/2} \Big]_1^4$$

$$= \frac{4\pi}{5}\left((32) - 1\right)$$

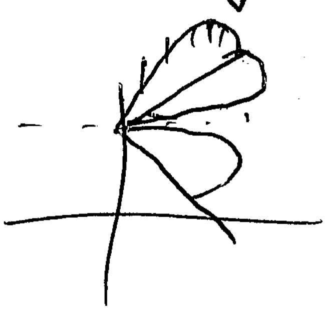
3. Consider the region bounded by the following curves. Hint: One point where they intersect is (8,2).
   a. (4 points) Carefully draw and label the region.

$x = 6y - y^2$ $\quad$ $x = 12 - 2y$

$x + 2y = 12$

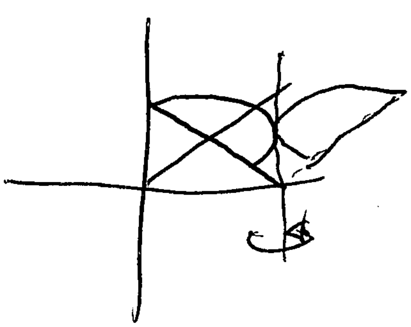


   b. (6 points) Set up the integral (but do not evaluate it) to find the volume of the solid obtained by rotating the region around the line y = 6.



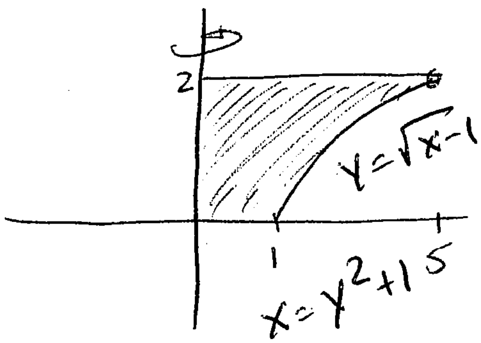
$$V = \int_2^6 2\pi(6-y)\left((6y-y^2)-(12-2y)\right)dy$$

   c. (6 points) Set up the integral (but do not evaluate it) to find the volume of the solid obtained by rotating the region around the line x = 9.



$$V = \int_2^6 \pi(12-2y)^2 - \pi(6y-y^2)^2\,dy$$

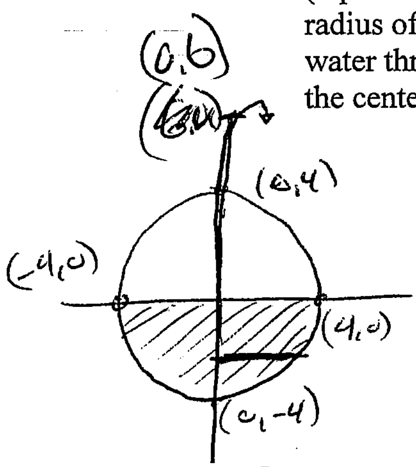
4. (8 points) Consider the graph $y = \sqrt{x-1}$ between x=1 and x= 5. Draw the region. Rotate this curve around the line y-axis to generate a volume. Suppose this volume is filled with water. Set up an integral to find the work required to pump all of the water out the top of the tank. <u>*Do not evaluate the integral.*</u> Suppose the x and y units are in meters and the density of water is 1000 kg per cubic meter.



$$W = \int_0^2 F \cdot \text{distance}$$

$$= \int_0^2 \underbrace{\pi(y^2+1)^2}_{\text{ar ea}} \cdot \underbrace{(1000)}_{\text{density}}\underbrace{(9.8)}_{\text{gravity}} \cdot \underbrace{(2-y)}_{\text{distance}} \underbrace{dy}_{\text{thickness}}$$

5. (8 points) Suppose there is a spherical tank filled half way with water that has a radius of 4 meters. Set up an integral to determine the work required to pump the water through a pipe to a height 2 meters above the tank. For consistency, suppose the center of the sphere is positioned at the origin.



$$W = \int_{-4}^{0} F \cdot (\text{distance})$$

$x^2+y^2=4^2$

$x=\sqrt{16-y^2}$

$$= \int_{-4}^{0} \pi\left(\sqrt{16-y^2}\right)^2 (1000)(9.8)(6-y)\,dy$$

6. (8 points) A force of 15 N is required to hold a spring (with natural length 20 cm) at a length of 30 cm. Find the work required to stretch the spring from its natural length to 33 cm.

$F = k \cdot x$

$15 = k \cdot (10)$

$k = 1.5$

$$W = \int_0^{13} k \cdot x \, dx$$

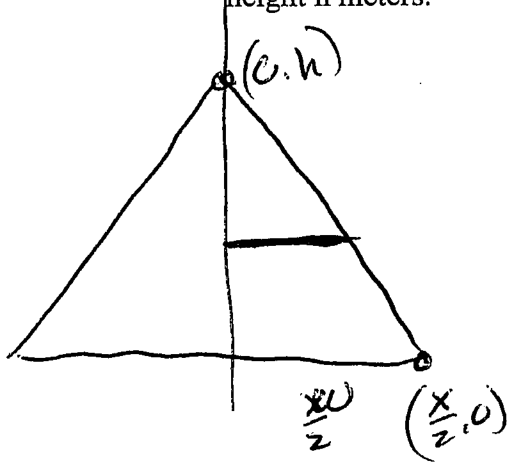
$$= \int_0^{13} 1.5 x \, dx$$

$$= \cancel{\int_0^{13}} \left. \frac{1.5 x^2}{2} \right]_0^{13}$$

$$= \frac{1.5}{2}\left(13^2 - 0^2\right)$$

7. (8 points) Find the volume [use calculus (only 1 point if you just write down the formula)] of a square-based pyramid with base x meters on each side meters and height h meters.

w

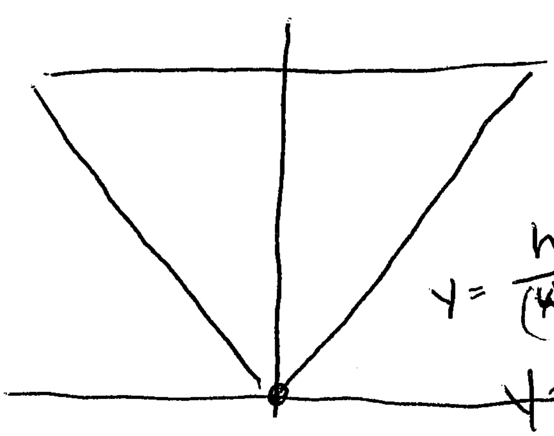


$$V = \int_0^h \underbrace{\left(2 \frac{w}{2h} y\right)^2}_{\text{area of cross sectn}} dy$$

$$= \int_0^h \frac{w^2}{h^2} y^2 \, dy$$

$$= \cancel{\int} \left. \frac{w^2}{h^2} \frac{y^3}{3} \right]_0^h =$$

$$\frac{1}{3} w^2 \cdot h$$



$y = \frac{h}{(w/2)} \cdot x$

$y = \frac{2h}{w} x$

$x = \frac{w}{2h} y$