Counting Cycle Decompositions of 2-regular Digraphs

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Abstract

A 2-regular digraph has indegree and outdegree 2 at every vertex. Loops and double arcs are allowed. It is elementary that every such digraph has a cycle decomposition, i.e., a partition of the arcs into directed cycles. Many 2-regular digraphs have more than one cycle decomposition. We show how to count the total number of cycle decompositions of all labeled 2-regular digraphs of given order n. The count can be also be obtained when the digraphs are restricted to having no loops or no double arcs.