Show all work!! Unsupported answers might not receive full credit.

1) Find \( \frac{dy}{dx} \).

\[ \cos(xy) + y = \sin(x - y) \]

\[
\frac{d}{dx} \left[ \cos(xy) + y \right] = \frac{d}{dx} \left[ \sin(x - y) \right] \\
\text{sum rule} \\
\frac{d}{dx} \cos(xy) + \frac{d}{dx} y = \cos(x-y) \frac{d}{dx} (x-y) \\
\text{chain rule} \\
-\sin(xy) \frac{d}{dx} [xy] + \frac{dy}{dx} = \cos(x-y) \left[ \frac{dx}{dx} - \frac{dy}{dx} \right] \\
\text{product rule} \\
-\sin(xy) \left[ (\frac{dx}{dx}) y + x (\frac{dy}{dx}) \right] + \frac{dy}{dx} = \cos(x-y) \left( 1 - \frac{dy}{dx} \right) \\
-\sin(xy) \left( 1 \cdot y + x \frac{dy}{dx} \right) + \frac{dy}{dx} = \cos(x-y) - \cos(x-y) \frac{dy}{dx} \\
-ysin(xy) - xsin(xy) \frac{dy}{dx} + \frac{dy}{dx} = \cos(x-y) - \cos(x-y) \frac{dy}{dx} \\
\frac{dy}{dx} = \cos(x-y) - xsin(xy) \frac{dy}{dx} + \frac{dy}{dx} = \cos(x-y) - \cos(x-y) \frac{dy}{dx} \\
\frac{dy}{dx} = \frac{\cos(x-y) + ysin(xy)}{\cos(x-y) - xsin(xy) + 1} \\
\frac{dy}{dx} = \frac{\cos(x-y) + ysin(xy)}{\cos(x-y) - xsin(xy) + 1} \\
\]

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\]
2) A boat is pulled into a dock by a rope attached to the bow of the boat that passes through a pulley on the dock that is 2 meters higher than the bow of the boat. If the rope is pulled in at a rate of 0.5 m/sec, how fast is the boat approaching the dock when it is 10 meters from the dock?

\begin{align*}
\frac{dw}{dt} &= -0.5 \\
\text{w is decreasing}
\end{align*}

\begin{align*}
x^2 + 2^2 &= w^2 \quad (1) \\
\frac{d}{dt}[x^2 + 4] &= \frac{d}{dt}[w^2] \\
2x \frac{dx}{dt} &= 2w \frac{dw}{dt} \\
\therefore \frac{dx}{dt} &= \frac{w}{x} \frac{dw}{dt}
\end{align*}

When \( x = 10 \), by (1) \( 10^2 + 2^2 = w^2 \), so \( w = \sqrt{104} \)

Then \( \frac{dx}{dt} \bigg|_{x=10, w=\sqrt{104}} = \left(-\frac{\sqrt{104}}{10} \right)(-0.5) \text{ m/s} = \frac{2\sqrt{26}}{20} \text{ m/s} = \frac{\sqrt{26}}{10} \text{ m/s} \)

**Bonus Problem**

(4 points)

Suppose that \( 1 \leq f'(x) \leq 4 \) for all values of \( x \). Show that \( 6 \leq f(10) - f(4) \leq 24 \).

\( \overline{f'} \) exists for all \( x \), so \( f \) is differentiable on \( (-\infty, \infty) \) and hence \( f \) is also continuous on \( (-\infty, \infty) \).

In particular, \( f \) is continuous on \([4,10)\) and \( f \) is differentiable on \((4,10)\). Thus, by the Mean Value Theorem, there is some \( c \) in \((4,10)\) such that \( f'(c) = \frac{f(10) - f(4)}{10 - 4} \)

Given \( 1 \leq f'(c) \leq 4 \), thus \( 1 \leq \frac{f(10) - f(4)}{6} \leq 4 \), so \( 6 \leq f(10) - f(4) \leq 24 \).