1. Let \( f(x) = x^3 - 9x^2 + 4x + 8 \).
   \[ f'(x) = 3x^2 - 18x + 4 \]
   \[ f''(x) = 6x - 18 \]
   a) [3 pts] Use interval notation to indicate where \( f(x) \) is increasing.
   \[ f'(0) = 0 \rightarrow 0 = 3x^2 - 18x + 4 \]
   \[ x = \frac{18 \pm \sqrt{(18)^2 - 4(3)(4)}}{2(3)} = 3 \pm \frac{1}{3}\sqrt{69} \]
   \[ f'(x) \text{ undefined } \rightarrow \text{ never} \]
   \[ x \in \left(3 - \frac{1}{3}\sqrt{69}, 3 + \frac{1}{3}\sqrt{69}\right) \]
   b) [3 pts] Use interval notation to indicate where \( f(x) \) is decreasing.
   a) \( f \) is increasing on \((-\infty, 3 - \frac{1}{3}\sqrt{69}) \cup (3 + \frac{1}{3}\sqrt{69}, \infty)\)
   b) \( f \) is decreasing on \( (3 - \frac{1}{3}\sqrt{69}, 3 + \frac{1}{3}\sqrt{69}) \)
   c) [3 pts] Use interval notation to indicate where \( f(x) \) is concave up.
   \[ f''(3) = 0 \rightarrow (6x - 18 = 0) \quad x = 3 \]
   \[ f''(x) \text{ undefined } \rightarrow \text{ never} \]
   \[ x = 3 \]
   d) [3 pts] Use interval notation to indicate where \( f(x) \) is concave down.
   \[ f \text{ is concave up on } (3, \infty) \]
   \[ f \text{ is concave down on } (-\infty, 3) \]
2. Consider the function \( f(x) = 4x^3 - 5x \) on the interval \([-4, 4]\).

a) [4 pts] Find the average or mean slope of the function on this interval.

\[
\frac{f(4) - f(-4)}{4 - (-4)} = \frac{(4(4)^3 - 5(4)) - (4(-4)^3 - 5(-4))}{8}
\]

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b) [4 pts] By the Mean Value Theorem, we know there exists at least one \( c \) in the open interval \((-4, 4)\) such that \( f'(c) \) is equal to this mean slope. For this problem, there are two values of \( c \) that work. Find both.

\[ f'(x) = 12x^2 - 5 \]

Find \( c \) such that

\[ f'(c) = 59 \]

\[ 12c^2 - 5 = 59 \]

\[ 12c^2 = 64 \]

\[ c^2 = \frac{64}{12} \]

\[ c = \pm \sqrt{\frac{16}{3}} \]

smaller one: \(-\frac{4}{\sqrt{3}}\), larger one: \(\frac{4}{\sqrt{3}}\)