PROBLEM 1. (10 points) A force of 8 lb is required to hold a spring stretched 9 in. beyond its natural length. How much work is done in stretching it from its natural length to 11 in beyond its natural length? (Express your answer in ft-lb.)

\[ \text{spring } F = kx \]

Given when \( F = 8 \text{ lbs} \), \( x = 9 \text{ in} = 0.75 \text{ ft} \)

\[ 8 \text{ lbs} = k \cdot (0.75 \text{ ft}) \]

\[ k = \frac{32}{3} \text{ lb-ft} \]

Thus, \( F = \frac{32}{3}x \).

\[ \Delta W = F \cdot \Delta x = \frac{32}{3}x \Delta x \]

\[ W = \sum \frac{32}{3}x \Delta x \]

Use \( x \) in feet:

\[ W = \int_{0}^{11/12} \frac{32}{3}x \, dx \]

\[ W = \frac{32}{3} \left. \frac{x^2}{2} \right|_{0}^{11/12} = \frac{16}{3} \left( \frac{11}{12} \right)^2 - 0 = \frac{121}{27} \text{ ft-lb} \]

FORM B ANS: 6 ft-lbs
PROBLEM 2. (10 points) Find the following integral

$$\int q^2 \ln (q^6) \, dq$$

Easiest to rewrite using the rule $\ln (a^n) = n \cdot \ln a$

$$\int q^2 \ln (q^6) \, dq = \int q^2 \cdot 6 \ln q \, dq = 6 \int q^2 \ln q \, dq$$

Then do integration by parts. $u = \ln q$, $dw = \frac{1}{q} \, dq$
$du = \frac{1}{q} \, dq$, $v = q^3$

$$\int q^2 \ln q \, dq = (\ln q) \cdot (q^3) - \int q^3 \frac{1}{q} \, dq$$

$$= q^3 \ln q - \int q^2 \, dq = \left[ 2q^3 \ln q - \frac{2}{3} q^3 \right] + C$$

You could also do parts from the beginning.

$u = \ln (q^6)$, $du = \frac{1}{q^6} \cdot 6q^5 \, dq$
$dv = q^2 \, dq$, $v = \frac{3}{2} q^3$

etc... try it both ways.

FORM B ANSWER: $\frac{3}{2} q^4 \ln q - \frac{3}{8} q^4 + C$