I. Put the following equation of a parabola into standard form. Then, graph the parabola. Make sure to label the focus, vertex, and directrix.

\[ y^2 - 4y - 4x = 0 \]

Then, graph the parabola. Make sure to label the focus, vertex, and directrix.

\[ (y - 2)^2 = 4x + 4 \]
\[ (y - 2)^2 = 4(x + 1) \]
\[ k = 2, h = -1, p = 1 \]

Vertex \((-1, 2)\)
Focus \((0, 2)\)
Directrix \(x = -2\)

II. Consider the following set of parametric equations.

a) Carefully sketch the curve on the given axis. Indicate the direction of increasing theta with an arrow.

\[ y = 2\cos \theta \quad x = 3\sin \theta \]

b) Eliminate the parameter to find a Cartesian equation of the curve.

\[ \left(\frac{y}{2}\right)^2 + \left(\frac{x}{3}\right)^2 = 1 \]

\[ \frac{y^2}{4} + \frac{x^2}{9} = 1 \]
III. Determine the points \((x, y)\) where the following parametric curve has horizontal and vertical tangents.

\[ y = t^3 - 3t \quad x = 1 - t \]

\[
\frac{dy}{dt} = 3t^2 - 3 \quad \frac{dx}{dt} = -1
\]

\[
\frac{dy}{dx} = \frac{3t^2 - 3}{-1} = -3t^2 + 3
\]

Fluid when \(\frac{dy}{dx} = 0\)

\[-3t^2 + 3 = 0 \quad -3(t^2 - 1) = 0 \quad -3(t+1)(t-1) = 0 \]

\[ t = -1, 1 \]

\[
\begin{array}{c|c|c}
 t & x & y \\
\hline
 -1 & 2 & 2 \\
 1 & 0 & -2 \\
\end{array}
\]

Horizontal tangents at \((2, 2)\) and \((0, -2)\)

\[
\frac{dx}{dt} \text{ is never zero so NO vertical tangents}
\]

IV. Determine the \(t\) intervals where the following parametric curve is concave up and concave down.

\[ y = 2t + \ln t \quad x = 2t - \ln t \quad \text{[NOTE! Domain is (0, \infty)]} \]

\[
\frac{dy}{dt} = 2 + \frac{1}{t} \quad \frac{dx}{dt} = 2 - \frac{1}{t}
\]

\[
\frac{dy}{dx} = \frac{2 + \frac{1}{t}}{2 - \frac{1}{t}} = \frac{2t + 1}{2t - 1}
\]

\[
\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2(2t-1) - 2(2t+1)}{(2t-1)^2} = \frac{-4}{(2t-1)^2}
\]

\[
\frac{d^2y}{dx^2} = \frac{-4(2t-1)^2}{2 - \frac{1}{t}} = \frac{-4t}{(2t-1)^3}
\]

When is \(\frac{d^2y}{dx^2} > 0, < 0, = 0\)?

- \(-\frac{4t}{(2t-1)^3}\) is undefined at \(t = \frac{1}{2}\)
- \(-\frac{4t}{(2t-1)^3}\) is zero when \(-\frac{4t}{(2t-1)^3} = 0\), \(t = 0\)

Concave up on \((0, \frac{1}{2})\)

Concave down on \((\frac{1}{2}, \infty)\)

V. Set up, but DO NOT INTEGRATE, an integral that represents the arc length of the following curve on the given interval.

\[ y = 2t - t^2 \quad x = 2t^{\frac{3}{2}} \quad 0 \leq t \leq 5 \]

\[
\frac{dy}{dt} = 2 - 2t \quad \frac{dx}{dt} = 3t^{\frac{1}{2}}
\]

\[ \int_{0}^{5} \sqrt{(3t^{\frac{1}{2}})^2 + (2-2t)^2} \, dt \]
VI. Convert the point \((x, y) = (-2, -3)\) to polar coordinates.

\[
\begin{align*}
  r^2 &= (-2)^2 + (-3)^2 = 13 \\
  r &= \pm \sqrt{13} \\
  \tan \theta &= -\frac{3}{2} \\
  \theta &= \tan^{-1}\left(\frac{3}{2}\right) \approx 0.98 \text{ radians} \\

\end{align*}
\]

\((\text{need } \pi + 0.98 \text{ with } r = \sqrt{13}) \text{ or } (-\sqrt{13}, -0.98)\)

Convert the point \((r, \theta) = \left(4, \frac{\pi}{3}\right)\) to rectangular coordinates.

\[
\begin{align*}
  x &= 4 \cos \left(\frac{\pi}{3}\right) = 2 \\
  y &= 4 \sin \left(\frac{\pi}{3}\right) = 2\sqrt{3}
\end{align*}
\]

\((2, 2\sqrt{3})\)

Convert the following polar equation to a rectangular equation.

\[
r = \sin \theta
\]

\[
\begin{align*}
  r^2 &= r \sin \theta \\
  x^2 + y^2 &= y
\end{align*}
\]

VII. Carefully graph the following polar curve.

Then, set up, but DO NOT INTEGRATE the integral(s) that represent the area between the loops.

\[
r = 1 + 2 \cos \theta
\]

\[
r = 0 \text{ when } 1 + 2 \cos \theta = 0 \\
\cos \theta = -\frac{1}{2} \\
\theta = \frac{2\pi}{3}, \frac{4\pi}{3}
\]

\[
\frac{\sqrt{3}}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} [1 + 2 \cos \theta]^2 \, d\theta - \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} [1 + 2 \cos \theta]^2 \, d\theta
\]

\[
\frac{1}{2} \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} [1 + 2 \cos \theta]^2 \, d\theta - 2 \left[ \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} [1 + 2 \cos \theta]^2 \, d\theta \right]
\]
VIII. Find the area of one petal of \( r = \cos 2\theta \)

\[
\frac{1}{2} \int_{-\pi/4}^{\pi/4} \left( \cos(2\theta) \right)^2 d\theta = \frac{1}{2} \int_{-\pi/4}^{\pi/4} \frac{1 + \cos(4\theta)}{2} d\theta
\]

\[
= \frac{1}{2} \left[ \frac{1}{2} \theta + \frac{1}{8} \sin(4\theta) \right]_{-\pi/4}^{\pi/4} = \frac{1}{2} \left[ \frac{\pi}{4} \right] = \frac{\pi}{8}
\]

IX. Find the value of \( \frac{dy}{dx} \) at \( \theta = \frac{\pi}{3} \) for the following polar curve: \( r = 3\sin \theta \)

\[
x = r\cos \theta = 3\sin \theta \cos \theta
\]

\[
y = r\sin \theta = 3\sin^2 \theta
\]

\[
\frac{dx}{d\theta} = 3\cos^2 \theta - 3\sin^2 \theta
\]

\[
\frac{dy}{d\theta} = 6\sin \theta \cos \theta
\]

\[
\frac{dy}{dx} = \frac{6\sin \theta \cos \theta}{3(\cos^2 \theta - \sin^2 \theta)}
\]

\[
\left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{3}} = \frac{6 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}}{3 \left( \frac{1}{4} - \frac{3}{4} \right)} = -\sqrt{3}
\]
X. Consider \( \mathbf{u} = \langle 3, -2 \rangle \) and \( \mathbf{v} = -7i - 5j \)

a) Find \( 3\mathbf{u} + \mathbf{v} \)

\[
3\langle 3, -2 \rangle + \langle -7, -5 \rangle = \langle 9, -6 \rangle + \langle -7, -5 \rangle = \langle 2, -11 \rangle
\]

b) Find \( \|\mathbf{u}\| \) (the norm or length of \( \mathbf{u} \))

\[
\|\mathbf{u}\| = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}
\]

c) Find a unit vector in the direction of \( \mathbf{u} \).

\[
\frac{1}{\|\mathbf{u}\|} \mathbf{u} = \frac{1}{\sqrt{13}} \langle 3, -2 \rangle = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle
\]

d) Find a vector \( \mathbf{x} \) such that \( 3\mathbf{u} - 2\mathbf{v} = \mathbf{x} - 2\mathbf{u} \)

\[
3\langle 3, -2 \rangle - 2\langle -7, -5 \rangle = \langle x_1, x_2 \rangle - 2\langle 3, -2 \rangle
\]

\[
\langle 9, -6 \rangle - \langle 14, -10 \rangle = \langle x_1, x_2 \rangle - \langle 6, -4 \rangle
\]

\[
\langle 9 + 14, -6 + 10 \rangle = \langle x_1 - 6, x_2 - 4 \rangle
\]

So, \( 23 = x_1 - 6 \) and \( 4 = x_2 + 4 \)

So, \( x_1 = 29 \) and \( x_2 = 0 \)

\( \mathbf{x} = \langle 29, 0 \rangle \)

XI. Suppose \( \|\mathbf{u}\| = 6 \) and the vector \( \mathbf{u} \) makes an angle of 120 degrees with the positive x axis. Write vector \( \mathbf{u} \) in component form.

\[
\mathbf{u} = 6 \cos(120^\circ) \hat{i} + 6 \sin(120^\circ) \hat{j}
\]

\[
\mathbf{u} = 6 \left( -\frac{1}{2} \right) \hat{i} + 6 \left( \frac{\sqrt{3}}{2} \right) \hat{j}
\]

\[
\mathbf{u} = -3 \hat{i} + 3\sqrt{3} \hat{j}
\]