1) Given $f(x, y) = x^2 + 9y^2$

a) Find the differential $df$.

b) Use the differential to approximate $f(2.1, -1.2) - f(2, -1)$.

c) Find the gradient $\nabla f(x, y)$.

d) Find the directional derivative of the function $f(x, y)$ at the point $(-2, 3)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

e) Find the maximal value of the directional derivative of $f$ at the point $(-2, 3)$. 
f) Find an equation of the tangent plane to the surface \( z = f(x, y) \) at the point \((-2, 3)\).

g) Consider the level curve \( 9 = x^2 + 9y^2 \). Using \( x(t) = 3\cos t \), \( y(t) = \sin t \), \( 0 \leq t < 2\pi \) as a parametrization of the level curve, show that \( f(x(t), y(t)) \) is a constant by substitution and simplifying in terms of \( t \).

h) From part g, show that \( \frac{df}{dt} = 0 \) using the chain rule, again showing that \( f(x(t), y(t)) \) is a constant.
2) Given \( f(x, y) = \sin(x^2 - y) \), \( x(u, v) = v \cos u \), and \( y(u, v) = -v^2 \sin u \). Compute and simplify the following partials using the chain rule.

a) \( \frac{\partial f}{\partial v} \)

b) \( \frac{\partial f}{\partial u} \)
3) Use implicit differentiation to find the first partials of $z$.

$$z^3 - x \ln z + 3y^2x^4 = e^{y-z}$$