Math 254 ~ Parametric Equations and Vector Review

Line through \((x_1, y_1)\) and \((x_2, y_2)\):

\[
x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1)
\]

or

\[
r(t) = \langle x_1, y_1 \rangle + t \langle x_2 - x_1, y_2 - y_1 \rangle
\]

Circle radius \(r\) centered at \((h, k)\):

\[
x = h + r \cos t, \quad y = k + r \sin t
\]

or

\[
r(t) = \langle h, k \rangle + r \langle \cos t, \sin t \rangle
\]

Definition – Length of a Vector

The \textbf{length} of the vector \(\mathbf{v} = \langle v_1, v_2, v_3 \rangle\) is

\[
||\mathbf{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2}.
\]

Definition – Parallel Vectors

Two nonzero vectors \(\mathbf{u}\) and \(\mathbf{v}\) are \textbf{parallel} if there is some scalar \(c\) such that \(\mathbf{u} = c\mathbf{v}\).

Definition – Dot Product

The \textbf{dot product} of \(\mathbf{u} = \langle u_1, u_2, u_3 \rangle\) and \(\mathbf{v} = \langle v_1, v_2, v_3 \rangle\) is

\[
\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3
\]

\text{Note: } \mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2
Definition – Orthogonal Vectors

The vectors $u$ and $v$ are **orthogonal** if $u \cdot v = 0$.

Theorem – Angle Between Two Vectors

If $\theta$ is the angle between two nonzero vectors $u$ and $v$, then

$$\cos \theta = \frac{u \cdot v}{\|u\|\|v\|}$$

Definition – Cross Product of Two Vectors in Space

Let $u = u_1 \mathbf{i} + u_2 \mathbf{j} + u_3 \mathbf{k}$ and $v = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$ be vectors in space. The **cross product** of $u$ and $v$ is the vector

$$u \times v = (u_2 v_3 - u_3 v_2) \mathbf{i} - (u_1 v_3 - u_3 v_1) \mathbf{j} + (u_1 v_2 - u_2 v_1) \mathbf{k}$$

or

$$u \times v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Theorem – Parametric Equations of a Line in Space

A line $L$ parallel to the vector $v = \langle a, b, c \rangle$ and passing through the point $(x_1, y_1, z_1)$ is represented by the **parametric equations**

$$x = x_1 + at, \ y = y_1 + bt, \ \text{and} \ z = z_1 + ct$$

Theorem – Standard Equation of a Plane in Space

The plane containing the point $(x_1, y_1, z_1)$ and having a normal vector $n = \langle a, b, c \rangle$ can be represented, in **standard form**, by the equation

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$