Algebra Fundamentals

Order of Operations

1) Parentheses – Simplify expressions within grouping symbols first starting with the innermost set. [Note: To simplify expressions inside parentheses, follow the order of operations.] Some types of grouping symbols: ( ), [ ], { }.
   Special cases:
   - Absolute value bars are also considered to be grouping symbols; | | .
   - Also, if there is a fraction bar, then there are hidden parentheses!

   \[ \frac{A}{B} \]
   is an abbreviation for \((A) ÷ (B)\).
   So, if fraction bars are present, simplify the numerator and denominator separately before performing the division.

2) Exponents – Raise to powers or take roots. [Note: Roots are similar to exponents and in fact roots can be represented by exponential notation, for example \(\sqrt{a} = a^{\frac{1}{2}}\).]

3) Multiplication or Division – Multiply or Divide (in order from left to right.) [Note: Division is a special kind of multiplication; multiplying by the reciprocal.]
   Multiplication and Division are on the same level of priority. If more than one of these operations appear, work from left to right.

4) Addition or Subtraction – Add or Subtract (in order from left to right.) [Note: Subtraction is a special kind of addition; adding the additive inverse.] Addition and Subtraction are on the same level of priority. If more than one of these operations appear, work from left to right.

Useful way to memorize the order: PEMDAS - Please Excuse My Dear Aunt Sally.

Variable – A variable is a symbol (usually a letter such as \(x, y, z\), etc…) that represents a number. It is a place holder telling us that a numerical value is to be put in this place. Sometimes, a variable is used to indicate that any number is allowed in the spot. Such as in rules of algebra like \(a + b = b + a\).
Other times, the variable is used to indicate that some unknown number (or unknown numbers) is to be put in the spot. Such as in solving an equation like \(2x - 5 = 7\).

Constant – A numerical constant is a fixed numerical value (one that does not depend upon the value of any variables that may be occurring.) Special numerical constants have been given their own symbols such as 0, 1, 2, 3, 4, \(\frac{1}{2}\), and \(\pi\) (these are usually called simply “numbers”.)
Algebraic Expression – An algebraic expression is formed by numbers (constants) and variables by the operations of addition, subtraction, multiplication, division, raising to powers and/or taking roots.

Evaluating an Expression – This is the process of simplifying an expression to a number. If the expression involves variables, then in order to evaluate the expression we must be given values for the variables. When given values for the variables, we replace (substitute) the values for the corresponding variables. We then simplify the expression following the Order of Operations.

Form of an Expression

In order to handle expressions and use algebraic rules, we must be able to identify the kinds of expressions that we are dealing with. There are several Forms of Expressions; Constant, Variable, Sum, Difference, Product, Quotient, Exponential, Radical, and Absolute Value. There are other forms, such as Function Notation (of which there are many special function you may encounter later in math, like sine, cosine, and logarithmic.)

To identify the Form of an Expression, go through the order of operations as if you were simplifying the expression. If it helps, choose simple values (like 0, 1, or 2) for any variables that appear. Keep track of the operations that you perform. *The very last operation that you perform indicates the Form of the Expression!*

Moreover, you can isolate pieces of an expression and find the form of each piece using this technique.

It is vital to know the form of an expression that you are handling. Every time you wish to use an algebraic rule, you must first check that the expression has the same form as that part of the rule you are using.

Substitution

Every time we replace a part of an expression with a value or another expression, we are using the technique of Substitution. And, *every time a substitution is performed, parentheses are used!* We first remove the part of the expression that we are changing. We then use parentheses to hold that position. Finally, we substitute the new expression or number into those parentheses. The parentheses are necessary to maintain the Order of Operations in the original expression.

However, this does become very tedious. Sometimes, the parentheses will just be dropped because the Order of Operations will remain unaffected. In these cases we leave out the step of writing out the parentheses. But, there are times when the parentheses are needed. *Be Careful! You should always keep in mind that parentheses are to be used in substitution. Make sure that they would not affect the Order of Operations before you skip the step of writing the parentheses.*