SHOW ALL YOUR WORK! GOOD LUCK!

Let \( R \) be the region bounded by the curves \( y = 9 - x^2 \) and \( y = x^2 + 1 \) and the \( y \)-axis.

1. (8pts) Sketch \( R \) and find its area. (Hint: The curves cross at \( x = -2 \) and \( x = 2 \))

\[
A = \int_{-2}^{2} \left[ (9 - x^2) - (x^2 + 1) \right] \, dx
= \int_{0}^{2} (8 - 2x^2) \, dx
= 8x - \frac{2}{3}x^3 \bigg|_{0}^{2} = 16 - \frac{16}{3} = \frac{32}{3}
\]

2. (6pts) Set up (but do not evaluate) an integral to find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

\[
dV = \pi (R^2 - r^2) \, dx \\
R = (9 - x^2) - 0 = 9 - x^2 \\
r = (x^2 + 1) - 0 = x^2 + 1
\]

\[
V = \int_{0}^{2} \pi \left[ (9 - x^2)^2 - (x^2 + 1)^2 \right] \, dx
\]

3. (6pts) Set up (but do not evaluate) an integral to find the volume of the solid obtained by rotating \( R \) about the line \( x = 3 \).

\[
dV = 2\pi r \cdot h \cdot \, dx \\
r = 3 - x \\
h = (9 - x^2) - (x^2 + 1)
\]

\[
V = \int_{0}^{2} 2\pi (3-x)(8-2x^2) \, dx
\]