Instructions: Show all of your work and justify all steps. Name theorems where appropriate. Correct answers are not worth points without the accompanying work. Write legibly. No decimal answers will be accepted unless there are decimals in the problem. You have 15 minutes.

1.) [7 points] Let \( f(x, y) = \ln \sqrt{x^2 + y^2} - 1 \), where \( x = 2st \), and \( y = \frac{t}{s} \).

(a) Using the chain rule (14.5), find \( \frac{\partial f}{\partial t} \). Your answer should be in variables \( x, y, s, t \).

\[
\begin{align*}
\frac{\partial f}{\partial t} &= \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} \\
&= \left( \frac{1}{2} \cdot \frac{2x}{x^2 + y^2 - 1} \right) \cdot 2s + \left( \frac{1}{2} \cdot \frac{2y}{x^2 + y^2 - 1} \right) \cdot \left( -\frac{s}{t^2} \right)
\end{align*}
\]

(b) Using part (a), evaluate \( \frac{\partial f}{\partial t} \) when \( s = 3 \) and \( t = 4 \).

\[
\begin{align*}
x(3, 4) &= 2.4 \\
y(3, 4) &= 3/4
\end{align*}
\]

\[
\begin{align*}
\frac{\partial f}{\partial t} \bigg|_{(x, y) = (3, 4)} &= \frac{2.4}{2.4^2 + (3/4)^2 - 1} \cdot 6 + \frac{3/4}{2.4^2 + (3/4)^2 - 1} \cdot \left( -\frac{3}{16} \right) \\
&= \frac{2201}{9209} - \frac{9/16}{9209/16} \\
&= \frac{2201}{9209} - \frac{9/16}{9209/16} = \frac{9207}{36,836} \approx 0.249946
\end{align*}
\]
2.) [8 points] Does the function \( f(x, t) = \sin(2x - 2at) + \cos(3x - 3at) \) satisfy the wave equation? Your solution should include all 4 relevant partial derivatives. (hint: Recall the wave equation is: \( \frac{\partial^2 f}{\partial t^2} = a^2 \frac{\partial^2 f}{\partial x^2} \))

\[
\frac{\partial f}{\partial t} = -2a \cos(2x - 2at) + 3a \sin(3x - 3at)
\]

\[
\frac{\partial^2 f}{\partial t^2} = -4a^2 \sin(2x - 2at) - 9a^2 \cos(3x - 3at)
\]

\[
\frac{\partial f}{\partial x} = 2 \cos(2x - 2at) - 3 \sin(3x - 3at)
\]

\[
\frac{\partial^2 f}{\partial x^2} = -4 \sin(2x - 2at) - 9 \cos(3x - 3at)
\]

\[
\frac{\partial^2 f}{\partial t^2} = a^2 \left( -4 \sin(2x - 2at) - 9 \cos(3x - 3at) \right)
\]

\[
= a^2 \frac{\partial^2 f}{\partial x^2}
\]

\( f \) does satisfy the wave equation.

\[ \]

Just for Fun.) [0 points]

How do you catch a unique rabbit? Unique up on it.

How do you catch a tame rabbit? Tame way ... unique up on it!