1.) Consider the lamina bounded by the curves \( r = 1 + \sin \theta \) and \( r = 3 \sin \theta \). The density at any point is inversely proportional to its distance from the origin.

(a) \([2 \text{ points}]\) Find the density function, \( \rho \).

\[
\rho(x, y) = \frac{k}{\sqrt{x^2 + y^2}}, \quad \rho(r \cos \theta, r \sin \theta) = \frac{k}{r}
\]

(b) \([3 \text{ points}]\) Set up, but do not evaluate, an iterated integral for the mass of the lamina.

\[
m = \iint \rho(x, y) \, dA = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{1+\sin \theta}^{3 \sin \theta} \frac{k}{r} \cdot r \, dr \, d\theta
\]

(c) \([4 \text{ points}]\) Set up, but do not evaluate, iterated integrals for the center of mass, \((\bar{x}, \bar{y})\).

\[
\bar{x} = \frac{1}{m} \cdot \iint_{R} x \rho(x, y) \, dA = \frac{1}{m} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{1+\sin \theta}^{3 \sin \theta} r \cos \theta \cdot \frac{k}{r} \cdot r \, dr \, d\theta
\]

\[
\bar{y} = \frac{1}{m} \cdot \iint_{R} y \rho(x, y) \, dA = \frac{1}{m} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{1+\sin \theta}^{3 \sin \theta} r \sin \theta \cdot \frac{k}{r} \cdot r \, dr \, d\theta
\]
2.) Consider the solid bounded by the parabolic cylinder \( x = y^2 \) and the planes \( z = 0, z = x \) and \( x = 1 \). Suppose the density at any point is proportional to its distance from the \( x \)-axis.

(a) [2 points] Find the density function, \( \rho \).
\[
\rho = k \sqrt{y^2 + z^2}
\]

(b) [4 points] Set up, but do not evaluate, a triple integral for the moment of inertia about the \( x \)-axis. Hint: recall the equation
\[
I_x = \iiint_E (x^2 + y^2) \rho(x, y, z) \, dV
\]

\[
\begin{align*}
I_x &= \iiint_E (x^2 + y^2) \cdot k \sqrt{y^2 + z^2} \, dz \, dx \, dy \\
I_x &= \int_0^1 \int_0^x \int_{-\sqrt{x}}^{\sqrt{x}} (x^2 + y^2) \cdot k \sqrt{y^2 + z^2} \, dz \, dx \, dy \\
I_x &= \int_0^1 \int_0^x (x^2 + y^2) \cdot k \sqrt{y^2 + z^2} \, dz \, dy \, dx \\
I_x &= \int_0^{\sqrt{2}} \int_0^1 (x^2 + y^2) \cdot k \sqrt{y^2 + z^2} \, dz \, dy \, dx
\end{align*}
\]

Remark: Integrating with respect to \( z \) first is clearly not the way to go here since the "back" of this solid is the plane \( x = z \) when \( z > y^2 \) and the parabolic cylinder when \( 0 < z < y^2 \).

See Exercise 15.6.34 for a similar example.