Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

1. A function f and a point P are given. Let θ correspond to the direction of the directional derivative. Complete parts **a.** through **e.**

$$f(x,y) = \sqrt{6 + x^2 + y^2}, \ P(1,1)$$

a. Find the gradient and evaluate it at P.

The gradient at P is $\langle \square , \square \rangle$.

(Type exact answers, using radicals as needed.)

b. Find the angles θ (with respect to the positive x-axis) associated with the directions of maximum increase, maximum decrease, and zero change. What angles are associated with the direction of maximum increase?

$\square$

(Type any angles in radians between 0 and 2π. Type an exact answer, using π as needed. Use a comma to separate answers as needed.)

What angles are associated with the direction of maximum decrease?

$\square$

(Type any angles in radians between 0 and 2π. Type an exact answer, using π as needed. Use a comma to separate answers as needed.)

What angles are associated with the direction of zero change?

$\square$

(Type any angles in radians between 0 and 2π. Type an exact answer, using π as needed. Use a comma to separate answers as needed.)

c. Write the directional derivative at P as a function of θ; call this function $g(\theta)$.

$g(\theta) = \square$

(Type an exact answer, using radicals as needed.)

d. Find the value of θ that maximizes $g(\theta)$ and find the maximum value. What value of θ maximizes $g(\theta)$?

$\theta = \square$

(Type any angles in radians between 0 and 2π. Type an exact answer, using π as needed.)

1.
(cont.)

What is the maximum value?

$g(\theta) = \square$

(Type an exact answer, using radicals as needed.)

e. Verify that the value of θ that maximizes g corresponds to the direction of the gradient. Verify that the maximum value of g equals the magnitude of the gradient. Are the values from part d consistent with the values from parts a and b?

- ○ Yes
- ○ No

Answers

$\frac{\sqrt{2}}{4}$

$\frac{\sqrt{2}}{4}$

$\frac{\pi}{4}$

$\frac{5\pi}{4}$

$\frac{3\pi}{4}, \frac{7\pi}{4}$

$\frac{\sqrt{2}}{4} \cos \theta + \frac{\sqrt{2}}{4} \sin \theta$

$\frac{\pi}{4}$

$\frac{1}{2}$

Yes

Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

2.

Consider the ellipsoid $f(x,y) = \sqrt{1 - \frac{x^2}{36} - \frac{y^2}{144}}$ and the point $P(-\sqrt{6},0)$ on the level curve $f(x,y) = \frac{5}{\sqrt{63}}$. Compute the slope of the line tangent to the level curve at P and verify that the tangent line is orthogonal to the gradient at that point.

What is the slope of the tangent line to the level curve at $P(-\sqrt{6},0)$? Select the correct choice below and, if necessary, fill in the answer box in your choice.

- A. The slope at $P(-\sqrt{6},0)$ is ☐.
- B. The slope at $P(-\sqrt{6},0)$ is undefined, so the tangent line is vertical.

How do you know the gradient and tangent line are orthogonal? Select the correct choice below and fill in the answer boxes in your choice.

- A. The tangent line slope is undefined, so it is a vertical line. The gradient $\langle$☐,☐$\rangle$ is horizontal, so it is orthogonal to the tangent line.
- B. The slope of the tangent line, m, can be used to write a vector in the direction of the tangent line, $\langle 1,m \rangle$. The dot product of this vector and the gradient $\langle$☐,☐$\rangle$ is 0, so they are orthogonal.

Answers B

A, $\frac{1}{6\sqrt{5}}$, 0

Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

3. Consider the function $f(x,y,z) = \frac{x-3z}{y-2z}$, the point $P(-1,-3,-3)$, and the unit vector $\mathbf{u} = \left\langle \frac{6}{11}, \frac{6}{11}, \frac{7}{11} \right\rangle$.

a. Compute the gradient of f and evaluate it at P.
b. Find the unit vector in the direction of maximum increase of f at P.
c. Find the rate of change of the function in the direction of maximum increase at P.
d. Find the directional derivative at P in the direction of the given vector.

---

a. What is the gradient at the point $(-1,-3,-3)$?

⟨☐,☐,☐⟩ (Simplify your answers.)

b. What is the unit vector in the direction of maximum increase?

⟨☐,☐,☐⟩
(Type exact answers, using radicals as needed.)

c. What is the rate of change in the direction of maximum increase?

☐ (Type an exact answer, using radicals as needed.)

d. What is the directional derivative in the direction of the given vector?

☐ (Type an exact answer, using radicals as needed.)

Answers $\frac{1}{3}$

$-\frac{8}{9}$

$\frac{7}{9}$

$\frac{3\sqrt{122}}{122}$

$-\frac{4\sqrt{122}}{61}$

3. (cont.)

$\frac{7\sqrt{122}}{122}$

$\frac{\sqrt{122}}{9}$

$\frac{19}{99}$

4. Consider the surface $f(x,y,z) = -x^2 - 4y^2 + 4z^2 + 4 = 0$, which may be regarded as a level surface of the function $w = f(x,y,z)$. The point $P(2,-1,-1)$ is on the surface.
a. Find the (three-dimensional) gradient of f and evaluate it at P.
b. The heads of all the vectors orthogonal to the gradient with their tails at P form a plane. Find an equation of that plane.

a. What is the gradient?

$\nabla f(2,-1,-1) = \langle \square, \square, \square \rangle$ (Simplify your answers.)

b. Choose the correct equation for the plane below.

- A. $-x - 4y + 4z = 0$
- B. $-x - 4y + 4z + 2 = 0$
- C. $-4x + 8y - 8z + 8 = 0$
- D. $-4x + 8y - 8z = 0$

Answers -4

8

-8

C

5. Find the critical points of the following function. Use the second derivative test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point. Confirm your results using a graphing utility.

$$f(x,y) = 3xy\,e^{-x^2-y^2}$$

What are the critical points?

☐

(Type an ordered pair. Use a comma to separate answers as needed.)

Identify any local maxima. Select the correct choice below and, if necessary, fill in the answer box within your choice.

○ A. There are local maxima at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)

○ B. There are no local maxima.

Identify any local minima. Select the correct choice below and, if necessary, fill in the answer box within your choice.

○ A. There are local minima at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)

○ B. There are no local minima.

Identify any saddle points. Select the correct choice below and, if necessary, fill in the answer box within your choice.

○ A. There are saddle points at ☐.
(Type an ordered pair. Use a comma to separate answers as needed.)

○ B. There are no saddle points.

Answers $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right),(0,0),\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right),\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

A, $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right),\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

5. (cont.)

A, $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right),\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

A, (0,0)

6. Find the point on the plane $5x + 4y + z = 12$ that is nearest to (2,0,1).

What are the values of x, y, and z for the point?

$x = \square$ $y = \square$ $z = \square$

(Type integers or simplified fractions.)

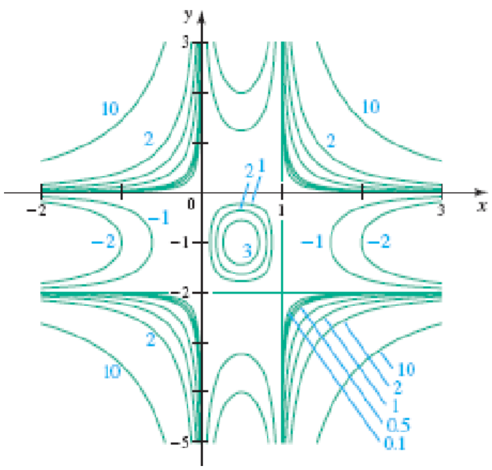
Answers $\frac{89}{42}$

$\frac{2}{21}$

$\frac{43}{42}$

Student: ____________________ **Date:** ______________________ **Time:** ______________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

7. Based on the level curves that are visible in the following graph, identify the approximate locations of the local maxima, local minima, and saddle points.



Identify the locations of the local maxima. Choose the correct answer below.

- A. There is a local maximum at $(0.5, -1)$.
- B. There are local maxima at $(1,0)$, $(1,-2)$, $(0,0)$, and $(0,-2)$.
- C. There are no local maxima.
- D. There is a local maximum at $(3,3)$.

Identify the locations of the local minima. Choose the correct answer below.

- A. There is a local minimum at $(2.5, -1)$.
- B. There are no local minima.
- C. There is a local minimum at $(0.5, -1)$.
- D. There are local minima at $(1,0)$, $(1,-2)$, $(0,0)$, and $(0,-2)$.

Identify the locations of the saddle points. Choose the correct answer below.

- A. There is a saddle point at $(0.5, -1)$.
- B. There are saddle points at $(1,0)$, $(1,-2)$, $(0,0)$, and $(0,-2)$.
- C. There are no saddle points.
- D. There is a saddle point at $(2.5, -1)$.

Answers A

B

B

8. Use the Second Derivative Test to prove that if (a,b) is a critical point of f at which $f_x(a,b) = f_y(a,b) = 0$ and $f_{xx}(a,b) < 0 < f_{yy}(a,b)$ or $f_{yy}(a,b) < 0 < f_{xx}(a,b)$, then f has a saddle point at (a,b).

The Second Derivative Test states that if $D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$, then f has a saddle point at (a,b) if

- $D(a,b) > 0.$
- $D(a,b) = 0.$
- $D(a,b) < 0.$

Regardless of the point (a,b),

- $(f_{xy}(a,b))^2 \ge 0.$
- $(f_{xy}(a,b))^2 \le 0.$
- $(f_{xy}(a,b))^2 = 0.$

If $f_{xx}(a,b) < 0 < f_{yy}(a,b)$, then

- $f_{xx}(a,b) f_{yy}(a,b) < 0.$
- $f_{xx}(a,b) f_{yy}(a,b) > 0.$
- $f_{xx}(a,b) + f_{yy}(a,b) < 0.$
- $f_{xx}(a,b) f_{yy}(a,b) = 0.$

Therefore,

- $D(a,b) > 0.$
- $D(a,b) < 0.$
- $D(a,b) = 0.$

If $f_{yy}(a,b) < 0 < f_{xx}(a,b)$, then

- $f_{xx}(a,b) f_{yy}(a,b) > 0.$
- $f_{xx}(a,b) f_{yy}(a,b) = 0.$
- $f_{xx}(a,b) + f_{yy}(a,b) < 0.$
- $f_{xx}(a,b) f_{yy}(a,b) < 0.$

Therefore,

- $D(a,b) < 0.$
- $D(a,b) > 0.$
- $D(a,b) = 0.$

Answers $D(a,b) < 0.$

$(f_{xy}(a,b))^2 \ge 0.$

$f_{xx}(a,b) f_{yy}(a,b) < 0.$

$D(a,b) < 0.$

$f_{xx}(a,b) f_{yy}(a,b) < 0.$

$D(a,b) < 0.$

Student: ____________________ **Date:** ______________________ **Time:** ______________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

9. Among all triangles with a perimeter of 45 units, find the dimensions of the triangle with the maximum area. It may be easiest to use Heron's formula, which states that the area of a triangle with side length a, b, and c is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where 2s is the perimeter of the triangle.

The dimensions are a = ☐ unit(s), b = ☐ unit(s), and c = ☐ unit(s). (Use ascending order.)

Answers 15

15

15

10. Let P be a plane tangent to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at a point in the first octant. Let T be the tetrahedron in the first octant bounded by P and the coordinate planes x = 0, y = 0, and z = 0. Find the minimum volume of T. (The volume of a tetrahedron is one-third the area of the base times the height.)

The minimum volume of T is ☐.

Answer: $\frac{abc\sqrt{3}}{2}$

11. Find the values of L and g with $L \geq 0$ and $g \geq 0$ that maximize the following utility function subject to the given constraint. Give the value of the utility function at the optimal point.

$$U = f(L,g) = 32L^{2/3}g^{1/3} \text{ subject to } 8L + 4g = 84$$

What are the values of L and g at the optimal point?

L = ☐, g = ☐

The value of the utility function at the optimal point is ☐.

Answers 7

7

224

12. Find the values of L and g with $L \geq 0$ and $g \geq 0$ that maximize the following utility function subject to the given constraint. Give the value of the utility function at the optimal point.

$$U = f(L,g) = L^{1/9}g^{8/9} \text{ subject to } 3L + 8g = 24$$

What are the values of L and g at the optimal point?

L = ☐, g = ☐

The value of the utility function at the optimal point is ☐.

Answers $\frac{8}{9}$

$\frac{8}{3}$

$\frac{8}{9} \cdot 3^{\frac{8}{9}}$

13. A lidless box is to be made using 588 square inches of cardboard. Find the dimensions of the box with the largest possible volume.

The dimensions of the box are ☐ inches.
(Type an exact answer in simplified form. Use a comma to separate answers as needed.)

Answer: 14,14,7

14. The temperature of points on an elliptical plate $x^2 + y^2 + xy \le 1$ is given by the equation $T(x,y) = 36(x^2 + y^2)$. Find the hottest and coldest temperatures on the edge of the elliptical plate.

Set up the equations that will be used by the method of Lagrange multipliers in two variables to solve this problem.

The constraint equation is ☐.

The vector equation is $\langle \square, \square \rangle = \lambda \langle \square, \square \rangle$.

The hottest temperature is ☐ degrees.

The coldest temperature is ☐ degrees.

Answers $x^2 + y^2 + xy - 1 = 0$

$72x$

$72y$

$2x + y$

$2y + x$

72

24

15. The paraboloid $z = 2x^2 + y^2 + 1$ and the plane $x - y + 2z = 4$ intersect in a curve C. Find the points on C that have maximum and minimum distance from the origin.

The point on C that is the maximum distance from the origin is (☐, ☐, ☐).
(Round to three decimal places as needed.)

The point on C that is the minimum distance from the origin is (☐, ☐, ☐).
(Round to three decimal places as needed.)

Answers -0.461

1.181

2.821

0.392

-0.498

1.555

Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

16. Suppose a thin rectangular plate, represented by a region R in the xy-plane, has a density given by the function $\rho(x,y)$; this function gives the area density in units such as g/cm^2. The mass of the plate is $\iint_R \rho(x,y)\ dA$. Assume that $R = \{(x,y): 0 \le x \le \frac{\pi}{2}, 0 \le y \le \pi\}$ and find the mass of the plates with the following density functions.

a. $\rho(x,y) = 11 + \sin x$
b. $\rho(x,y) = 11 + \sin y$
c. $\rho(x,y) = 11 + \sin x \sin y$

Set up the double integral as an iterated interval.

$$\iint_R \rho(x,y)\ dA = \int_{\square}^{\square}\int_{\square}^{\square} \rho(x,y)\ dx\ dy$$

a. The mass of the plate is ☐.
(Type an exact answer, using π as needed.)

b. The mass of the plate is ☐.
(Type an exact answer, using π as needed.)

c. The mass of the plate is ☐.
(Type an exact answer, using π as needed.)

Answers
0

π

0

$\frac{\pi}{2}$

$\frac{11}{2}\pi^2 + \pi$

$\frac{11}{2}\pi^2 + \pi$

$\frac{11}{2}\pi^2 + 2$

17. Let $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$.

a. Evaluate $\iint_R \cos(x\sqrt{y})\, dA$.

b. Evaluate $\iint_R x^3 y \cos(x^2y^2)\, dA$.

a. $\iint_R \cos(x\sqrt{y})\, dA = \square$

(Type an exact answer in simplified form. Use integers or fractions for any numbers in the expression.)

b. $\iint_R x^3 y \cos(x^2y^2)\, dA = \square$

(Type an exact answer in simplified form. Use integers or fractions for any numbers in the expression.)

Answers $2 - 2\cos 1$

$\frac{1}{4} - \frac{1}{4}\cos 1$

18. Evaluate the following integral as it is written.

$$\int_{\pi/4}^{\pi} \int_{\cos x}^{\sin x} dy\, dx$$

$\int_{\pi/4}^{\pi} \int_{\cos x}^{\sin x} dy\, dx = \square$

(Type an exact answer, using radicals as needed.)

Answer: $1 + \sqrt{2}$

19. Write an iterated integral of a continuous function f over the following region.

$$R = \{(x,y): 0 \le x \le y(11-y)\}$$

Choose the correct answer below.

A. $\int_0^{121} \int_{x^2-11}^{0} f(x,y)\, dy\, dx$

B. $\int_0^{11} \int_{x-11}^{x^2+11} f(x,y)\, dy\, dx$

C. $\int_{-11}^{11} \int_{y(11-y)}^{11} f(x,y)\, dx\, dy$

D. $\int_0^{11} \int_0^{y(11-y)} f(x,y)\, dx\, dy$

Answer: D

20. Evaluate the following integral as written.

$$\int_{-1}^{2} \int_{y}^{4-y} dx\, dy$$

$\int_{-1}^{2} \int_{y}^{4-y} dx\, dy = \square$ (Simplify your answer.)

Answer: 9

21. Reverse the order of integration in the following integral.

$$\int_0^1 \int_0^{\cos^{-1} y} f(x,y)\, dx\, dy$$

Reverse the order of integration.

$\int_{\square}^{\square} \int_{\square}^{\square} f(x,y)\, dy\, dx$

(Type exact answers, using π as needed.)

Answers 0

$\frac{\pi}{2}$

0

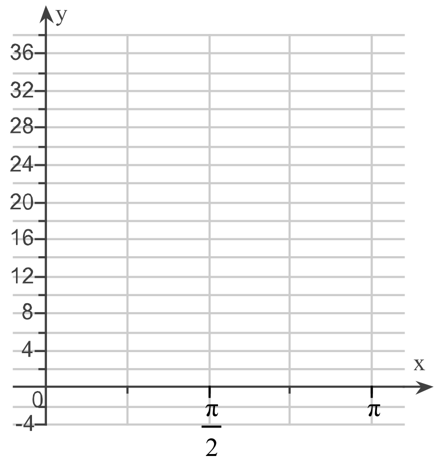
$\cos x$

Student: ____________________
Date: ____________________
Time: ____________________

Instructor: Hoi Nguyen
Course: AU14 MATH 2173 - ENG MATH B (20686) COL
Book: Ohio State University: Calculus: Early Transcendentals, Second Custom Edition

Assignment: Further practice problems for Midterm #1

22. Use a double integral to compute the area of the region bounded by $y = 18 + 18\,\mathbf{sin}\,x$ and $y = 18 - 18\,\mathbf{sin}\,x$ on the interval $[0,\pi]$. Make a sketch of the region.



Use the graphing tool to graph the boundary curves and shade the region.

Click to enlarge graph

Find the double integral needed to determine the area of the region.

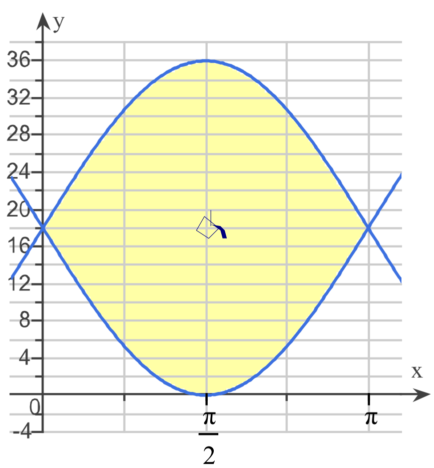
$\int_{\square}^{\square}\int_{\square}^{\square}(\square)\,dy\,dx$

(Type exact answers.)

The area of the region is $\square$

cubic units.
square units.
units.

(Simplify your answer.)



Answers

0

π

22.
(cont.)

$18 - 18\,\mathbf{sin}\ x$

$18 + 18\,\mathbf{sin}\ x$

1

72

square units.

23.

Under suitable conditions on f, $\int_a^{\infty}\int_{g(x)}^{h(x)} f(x,y)\ dy\ dx = \lim_{b \to \infty}\int_a^{b}\int_{g(x)}^{h(x)} f(x,y)\ dy\ dx$. Use or extend the one-variable methods for improper integrals to evaluate the integral below.

$$\int_1^{\infty}\int_0^{e^{-x}} xy^5\ dy\ dx$$

$\int_1^{\infty}\int_0^{e^{-x}} xy^5\ dy\ dx = \square$

(Type an exact answer.)

Answer: $\frac{7}{216} e^{-6}$

24. Compute the volume of the solid bounded by the planes below.

$$x = 0,\ x = 7,\ z = y - 2,\ z = -4y - 2,\ z = 0,\ z = 6$$

Find the double integral needed to determine the volume of the solid.

$$\square \int_{\square}^{\square} \int_{\square}^{\square} (\square)\, dz\, dx$$

The volume of the solid is $\square$ [cubic units. / square units. / units.] (Simplify your answer.)

Answers $\frac{5}{4}$

0

7

0

6

$z + 2$

$\frac{525}{2}$

cubic units.

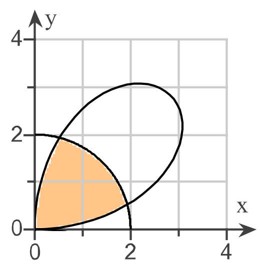
25.

Sketch the following region R. Then express $\iint_R f(r,\theta)\,dA$ as an iterated integral over R.

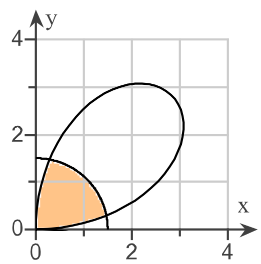
The region outside the circle $r = 2$ and inside the rose $r = 4\,\mathbf{sin}\,2\theta$ in the first quadrant.

Sketch the region R. Choose the correct graph below.

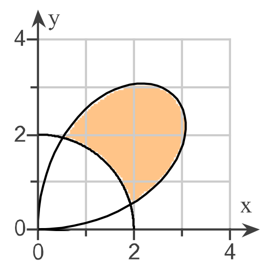
A.



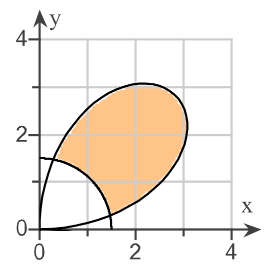
B.



C.



D.



Express $\iint_R f(r,\theta)\,dA$ as an iterated integral over R.

$$\int_{\square}^{\square}\int_{\square}^{\square} f(r,\theta)\,r\,dr\,d\theta$$

(Type exact answers. Type the coordinates for θ in radians between 0 and 2π.)

Answers C

$\frac{\pi}{12}$

$\frac{5\pi}{12}$

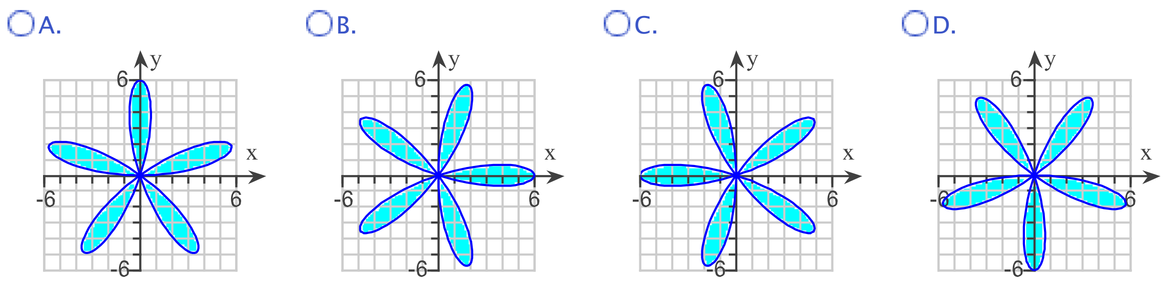
2

$4\,\mathbf{sin}\,(2\theta)$

26. Sketch the region and use a double integral to find its area.

The region bounded by all leaves of the rose $r = 6 \textbf{ sin } 5\theta$

Sketch the region. Choose the correct graph below.



Set up the double integral as efficiently as possible, in polar coordinates, that is used to find the area the leaf that is closest to the positive x-axis.

$$A_{leaf} = \int_{\square}^{\square} \int_{\square}^{\square} r \, dr \, d\theta$$

(Type exact answers, using π as needed.)

Find the area of the region.

$A_{rose} = \square$

units^2
units^3
units^4
units

(Type an exact answer, using π as needed.)

Answers A

0

$\frac{\pi}{5}$

0

$6 \textbf{ sin } 5\theta$

26. (cont.)

9π

units^2

27.

Find the average value of $\frac{1}{r^2}$ over the annulus $\{(r,\theta): 2 \le r \le 3\}$.

The average value is $\square$.
(Type an exact answer.)

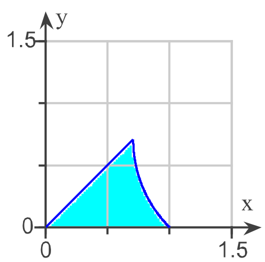
Answer: $\frac{2(\ln 3 - \ln 2)}{5}$

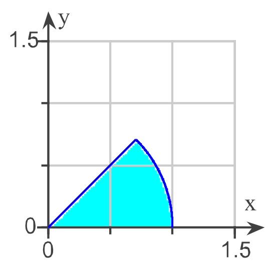
28.

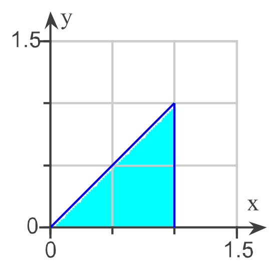
Sketch the region of integration and evaluate the following integrals, using the method of your choice.

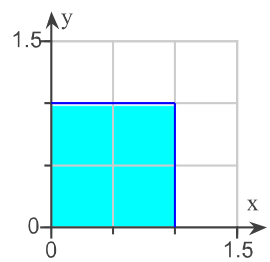
$$\int_0^{\frac{\pi}{4}} \int_0^{\sec\theta} 9r^3 \, dr\, d\theta$$

Sketch the region of integration. Choose the correct graph below.

A.


B.


C.


D.


Find the value of the double integral.

$$\int_0^{\frac{\pi}{4}} \int_0^{\sec\theta} 9r^3 \, dr\, d\theta = \square$$

(Type an exact answer, using π as needed.)

Answers C

3

Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

29. Suppose the density of a thin plate represented by the region R is $\rho(r,\theta)$ (in units of mass per area). The mass of the plate is $\iint_R \rho(r,\theta)\ dA$. Find the mass of the thin half annulus $R = \{(r,\theta): 1 \le r \le 4,\ 0 \le \theta \le \pi\}$ with a density $\rho(r,\theta) = 4 + r\sin\theta$.

Set up the double integral, in polar coordinates, that is used to find the mass.

$$\int_{\square}^{\square}\int_{\square}^{\square}(\square)\ dr\ d\theta$$

(Type exact answers. Use integers or fractions for any numbers in the expression.)

Find the mass.

$M = \square$

(Type an exact answer.)

Answers 0

π

1

4

$r(4 + r\sin\theta)$

$30\pi + 42$

Student: ____________________ **Date:** ____________________ **Time:** ____________________	**Instructor:** Hoi Nguyen **Course:** AU14 MATH 2173 - ENG MATH B (20686) COL **Book:** Ohio State University: Calculus: Early Transcendentals, Second Custom Edition	**Assignment:** Further practice problems for Midterm #1

30.

An important integral in statistics associated with the normal distribution is $I = \int_{-\infty}^{\infty} e^{-x^2} dx$. It is evaluated in the following steps.

a. Assume that $I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2} dy\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2} dxdy$, where we have chosen the variables of itegration to be x and y and then written the product as an iterated integral. Evaluate this integral in polar coordinates.

b. Evaluate $\int_0^{\infty} e^{-x^2} dx$, $\int_0^{\infty} x e^{-x^2} dx$, and $\int_0^{\infty} x^2 e^{-x^2} dx$ (using part (a) if needed).

---

a. $I^2 = \square$

(Type an exact answer, using π as needed.)

b. $\int_0^{\infty} e^{-x^2} dx = \square$

(Type an exact answer, using π as needed.)

$\int_0^{\infty} x e^{-x^2} dx = \square$

(Type an exact answer, using π as needed.)

$\int_0^{\infty} x^2 e^{-x^2} dx = \square$

(Type an exact answer, using π as needed.)

Answers π

$\frac{\sqrt{\pi}}{2}$

$\frac{1}{2}$

$\frac{\sqrt{\pi}}{4}$