The complex of p-centric and p-radical subgroups
and
its reduced Lefschetz module

John Maginnis and Silvia Onofrei*
Kansas State University and The Ohio State University

AMS Fall Central Sectional Meeting, University of Akron, Ohio, 20-21 October 2012
Subgroup Complexes in a Finite Group $G$

Subgroup complex $\Delta = \Delta(C)$

- 0-simplices: $C = \{Q : Q \leq G\}$ is a collection of subgroups of the group $G$, closed under $G$-conjugation and partially ordered by inclusion
- $n$-simplices: $\sigma = (Q_0 < Q_1 < \ldots < Q_n), Q_i \in C$

The group $G$ acts by conjugation on the subgroup complex $\Delta$:

- isotropy group of $\sigma$: $G_\sigma = \cap_{i=0}^n N_G(Q_i)$
- fixed point set of $Q$: $\Delta^Q = \Delta(C^Q)$ with $C^Q = \{P \in C | Q \leq N_G(P)\}$
The reduced Lefschetz virtual module with coefficients in a field $k$ of characteristic $p$

- alternating sum of chain groups:  
  $$\Lef_{G}(\Delta; k) := \sum_{i=-1}^{\left|\Delta\right|} (-1)^i C_i(\Delta; k)$$

- element of Green ring of $kG$:  
  $$\Lef_{G}(\Delta; k) = \sum_{\sigma \in \Delta/G} (-1)^{|\sigma|} \text{Ind}_{G_{\sigma}}^{G} k - k$$
The reduced Lefschetz virtual module with coefficients in a field $k$ of characteristic $p$

- alternating sum of chain groups: 
  \[ \tilde{L}_G(\Delta; k) := \sum_{i=-1}^{\vert \Delta \vert} (-1)^i C_i(\Delta; k) \]

- element of Green ring of $kG$:
  \[ \tilde{L}_G(\Delta; k) = \sum_{\sigma \in \Delta/G} (-1)^{\vert \sigma \vert} \text{Ind}^G_{G_{\sigma}} k - k \]

Theorem (Robinson, 1988)

*Let $G$ be a finite group, $k$ a field of characteristic $p$ and $\Delta$ a subgroup complex in $G$. The number of indecomposable summands of $\tilde{L}_G(\Delta; k)$ with vertex $Q$ equals the number of indecomposable summands of $\tilde{L}_{N_G(Q)}(\Delta^Q; k)$ with vertex $Q$.***
A nontrivial $p$-subgroup $Q$ of $G$ is $p$-radical if $Q = O_p(N_G(Q))$

is $p$-centric if $Z(Q) \in \text{Syl}_p(C_G(Q))$
The Complex of $p$-Centric and $p$-Radical Subgroups

- A nontrivial $p$-subgroup $Q$ of $G$ is $p$-radical if $Q = O_p(N_G(Q))$
  is $p$-centric if $Z(Q) \in \text{Syl}_p(C_G(Q))$

$D_p(G)$ complex
- collection of $p$-centric $p$-radical subgroups of $G$
- best candidate for a $p$-local geometry
- used in cohomology decompositions

$\widetilde{L}_G(D_p(G); k)$
- not indecomposable, not projective
- vertices are subgroups of non-centric $p$-radicals

Dwyer(1997)
Smith, Yoshiara(1997)
Dwyer(1998), Grodal(2001)
Benson, Smith(2008)
Sawabe(2005)
The Complex of $p$-Centric and $p$-Radical Subgroups

- A nontrivial $p$-subgroup $Q$ of $G$ is $p$-radical if $Q = O_p(N_G(Q))$
  is $p$-centric if $Z(Q) \in \text{Syl}_p(C_G(Q))$

$\mathcal{D}_p(G)$ complex
- collection of $p$-centric $p$-radical subgroups of $G$
- best candidate for a $p$-local geometry
- used in cohomology decompositions

$\tilde{L}_G(\mathcal{D}_p(G); k)$
- not indecomposable, not projective
- vertices are subgroups of non-centric $p$-radicals

- If $G$ is a finite simple group of Lie type then $\tilde{L}_G(\mathcal{D}_p(G); k) \simeq \text{St}_G$
  the irreducible and projective Steinberg module.
Terminology and Notation: Groups

- $G$ is a finite group and $p$ a prime divisor of its order
- a $p$-local subgroup is the normalizer of a finite $p$-subgroup of $G$
- a $p$-central element is an element in the center of a Sylow $p$-subgroup of $G$
- $kG$ is the group algebra with $k$ a field of characteristic $p$
Terminology and Notation: Groups

- $G$ is a finite group and $p$ a prime divisor of its order
- A $p$-local subgroup is the normalizer of a finite $p$-subgroup of $G$
- A $p$-central element is an element in the center of a Sylow $p$-subgroup of $G$
- $kG$ is the group algebra with $k$ a field of characteristic $p$

1. $G$ has characteristic $p$ if $C_G(O_p(G)) \leq O_p(G)$
2. $G$ has local characteristic $p$ if all $p$-local subgroups of $G$ have characteristic $p$
3. $G$ has parabolic characteristic $p$ if all $p$-local subgroups which contain a Sylow $p$-subgroup of $G$ have characteristic $p$
Proposition (Maginnis, Onofrei, 2009)

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(D_p(G); k)$ contains a conjugate of $t$. 

Sketch of proof:
Set $T := \langle t \rangle$. If $O_p(C_G(T))$ contains $p$-central elements then $D_p(G) \cap T$ is $N_G(T)$-contractible. Thus $D(G) \cap T$ is mod-$p$ acyclic. And P.A. Smith theory: $D(G) \cap Q$ is mod-$p$ acyclic for any $p$-subgroup $Q > T$. It follows $eL_N(G)(Q)(D(G) \cap Q; k) = 0$. An application of Robinson's theorem gives the result.
No Vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$. 
No Vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $\mathcal{D}_p(G)^T$ is $N_G(T)$-contractible.
No Vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $\mathcal{D}_p(G)^T$ is $N_G(T)$-contractible.
- Thus $\mathcal{D}(G)^T$ is mod-$p$ acyclic.
No Vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $\mathcal{D}_p(G)^T$ is $N_G(T)$-contractible.
- Thus $\mathcal{D}(G)^T$ is mod-$p$ acyclic.
- And P.A. Smith theory: $\mathcal{D}(G)^Q$ is mod-$p$ acyclic for any $p$-subgroup $Q > T$. 
No Vertex of $\tilde{L}_G(D_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\tilde{L}_G(D_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $D_p(G)^T$ is $N_G(T)$-contractible.
- Thus $D(G)^T$ is mod-$p$ acyclic.
- And P.A. Smith theory: $D(G)^Q$ is mod-$p$ acyclic for any $p$-subgroup $Q > T$.
- It follows $\tilde{L}_{N_G(Q)}(D(G)^Q; k) = 0$. 

An application of Robinson's theorem gives the result.
No Vertex of $\widetilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

Proposition (Maginnis, Onofrei, 2009)

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\widetilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

Sketch of proof:

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $\mathcal{D}_p(G)^T$ is $N_G(T)$-contractible.
- Thus $\mathcal{D}(G)^T$ is mod-$p$ acyclic.
- And P.A. Smith theory: $\mathcal{D}(G)^Q$ is mod-$p$ acyclic for any $p$-subgroup $Q > T$.
- It follows $\widetilde{L}_{N_G(Q)}(\mathcal{D}(G)^Q; k) = 0$.
- An application of Robinson’s theorem gives the result.
No Vertex of \( \tilde{L}_G(\mathcal{D}_p(G); k) \) Contains \( p \)-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume \( G \) is a finite group of parabolic characteristic \( p \). Suppose that \( t \) is an element of order \( p \) in \( G \) such that \( O_p(C_G(t)) \) contains \( p \)-central elements. Then no vertex of the reduced Lefschetz module \( \tilde{L}_G(\mathcal{D}_p(G); k) \) contains a conjugate of \( t \).

**Sketch of proof:**

- Set \( T := \langle t \rangle \).
- If \( O_p(C_G(T)) \) contains \( p \)-central elements then \( \mathcal{D}_p(G)^T \) is \( N_G(T) \)-contractible.
- Thus \( \mathcal{D}(G)^T \) is mod-\( p \) acyclic.
- And P.A. Smith theory: \( \mathcal{D}(G)^Q \) is mod-\( p \) acyclic for any \( p \)-subgroup \( Q > T \).
- It follows \( \tilde{L}_{N_G(Q)}(\mathcal{D}(G)^Q; k) = 0 \).
- An application of Robinson’s theorem gives the result.
No Vertex of $\widetilde{L}_G(\mathcal{D}_p(G); k)$ Contains $p$-Central Elements

**Proposition (Maginnis, Onofrei, 2009)**

Assume $G$ is a finite group of parabolic characteristic $p$. Suppose that $t$ is an element of order $p$ in $G$ such that $O_p(C_G(t))$ contains $p$-central elements. Then no vertex of the reduced Lefschetz module $\widetilde{L}_G(\mathcal{D}_p(G); k)$ contains a conjugate of $t$.

**Sketch of proof:**

- Set $T := \langle t \rangle$.
- If $O_p(C_G(T))$ contains $p$-central elements then $\mathcal{D}_p(G)^T$ is $N_G(T)$-contractible.
- Thus $\mathcal{D}(G)^T$ is mod-$p$ acyclic.
- And P.A. Smith theory: $\mathcal{D}(G)^Q$ is mod-$p$ acyclic for any $p$-subgroup $Q \supset T$.
- It follows $\widetilde{L}_{N_G(Q)}(\mathcal{D}(G)^Q; k) = 0$.
- An application of Robinson’s theorem gives the result.
Let $G$ be a finite group of parabolic characteristic $p$ and let $T$ be a $p$-subgroup of $G$. Assume the following hold:

(N1) The group $O_C := O_p(TC_G(T))$ is purely noncentral in $G$.

(N2) $C := TC_G(T) = O_C.H.K$ where $H$ has parabolic characteristic $p$ and $L := O_C.H$ is normal in $N_G(T)$.

(N3) A Sylow $p$-subgroup of $L$ contains $p$-central elements of $G$.

Then there is an $N_G(T)$-equivariant homotopy equivalence between $D_p(G)^T$ and $D_p(H)$.
Main Theorem on Vertices of $\widetilde{L}_G(D_p(G); k)$

**Theorem (Maginnis, Onofrei, 2012)**

Let $G$ be a finite group of parabolic characteristic $p$ and let $T$ be a $p$-subgroup of $G$. Assume that the following conditions hold:

1. \( C := TC_G(T) = O_C.H.K \) where \( O_C = O_p(C) \)
   and \( L := O_C.H \) is the generalized Fitting subgroup of $C$.

2. The group $H = L/O_C$ is a finite simple group of Lie type in characteristic $p$.


Then $T$ is a vertex of an indecomposable summand of $\widetilde{L}_G(D_p(G); k)$ if and only if:

1. $T = O_C$ is purely noncentral.
2. $K = C/L$ is a $p'$-group.
3. $|N_G(T)/C|$ is relatively prime to $p$.

Under these conditions, there will exist a unique indecomposable summand of $\widetilde{L}_G(D_p(G); k)$ with vertex $T$, which will lie in a block of $kG$ with defect group $T$. 
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\tilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\tilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\tilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\widetilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
- $\mathcal{D}_p(G)^T$ is $N_G(T)$-equivariantly homotopy equivalent to $\mathcal{D}_p(H)$
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\tilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\tilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
- $\mathcal{D}_p(G)^T$ is $N_G(T)$-equivariantly homotopy equivalent to $\mathcal{D}_p(H)$
- $\mathcal{D}_p(H)$ is $N_G(T)$-equivariantly homotopy equivalent to the Tits building $\Delta$ of $H$
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\widetilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(T C_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
- $\mathcal{D}_p(G)^T$ is $N_G(T)$-equivariantly homotopy equivalent to $\mathcal{D}_p(H)$
- $\mathcal{D}_p(H)$ is $N_G(T)$-equivariantly homotopy equivalent to the Tits building $\Delta$ of $H$
- $\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k) = M$ is an irreducible $kN_G(T)$-module, it is the inflation to $N_G(T)$ of the extended Steinberg module for $H.K'$

\[O_C = O_p(C) = O_C(T C_G(T))\]
\[N_G(T) = O_C.H.K' \text{ with } K' = N_G(T)/L \text{ and } L = O_C.H\]
\[\mathcal{D}_p(G)^T \text{ is } N_G(T)\text{-equivariantly homotopy equivalent to } \mathcal{D}_p(H)\]
\[\mathcal{D}_p(H) \text{ is } N_G(T)\text{-equivariantly homotopy equivalent to the Tits building } \Delta \text{ of } H\]
\[\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k) = M \text{ is an irreducible } kN_G(T)\text{-module, it is the inflation to } N_G(T) \text{ of the extended Steinberg module for } H.K'\]
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\widetilde{L}_G(\mathcal{D}_p(G); k)$ if and only if $T$ is a vertex of $\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
- $\mathcal{D}_p(G)^T$ is $N_G(T)$-equivariantly homotopy equivalent to $\mathcal{D}_p(H)$
- $\mathcal{D}_p(H)$ is $N_G(T)$-equivariantly homotopy equivalent to the Tits building $\Delta$ of $H$
- $\widetilde{L}_{N_G(T)}(\mathcal{D}_p(G)^T; k) = M$ is an irreducible $kN_G(T)$-module, it is the inflation to $N_G(T)$ of the extended Steinberg module for $H.K'$
- using that $kH$ has two blocks, the principal block and a block of defect zero, and that $L = F^*(C)$, the generalized Fitting subgroup of $C$, we deduce that $M$ lies in a nonprincipal block of $kN_G(T)$ with defect group $O_C.S' \simeq \nu x(M)$ and with $S'$ a Sylow $p$-subgroup of $K'$
Main Theorem: Sketch of the Argument

- $T$ is a vertex of $\tilde{L}_G(D_p(G); k)$ if and only if $T$ is a vertex of $\tilde{L}_{N_G(T)}(D_p(G)^T; k)$
- $O_C = O_p(C) = O_C(TC_G(T))$ is noncentral
- $N_G(T) = O_C.H.K'$ with $K' = N_G(T)/L$ and $L = O_C.H$
- $D_p(G)^T$ is $N_G(T)$-equivariantly homotopy equivalent to $D_p(H)$
- $D_p(H)$ is $N_G(T)$-equivariantly homotopy equivalent to the Tits building $\Delta$ of $H$
- $\tilde{L}_{N_G(T)}(D_p(G)^T; k) = M$ is an irreducible $kN_G(T)$-module, it is the inflation to $N_G(T)$ of the extended Steinberg module for $H.K'$
- using that $kH$ has two blocks, the principal block and a block of defect zero, and that $L = F^*(C)$, the generalized Fitting subgroup of $C$, we deduce that $M$ lies in a nonprincipal block of $kN_G(T)$ with defect group $O_C.S' \cong \nu x(M)$ and with $S'$ a Sylow $p$-subgroup of $K'$
- since $T \leq O_C$, we have $T \cong \nu x(M)$ if and only if $T = O_C$ and $S' = 1$
Examples Involving Sporadic Simple Groups in Characteristic 3

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fall Central Sectional Meeting, University of Akron, 20-21 October 2012
### Examples Involving Sporadic Simple Groups in Characteristic 3

<table>
<thead>
<tr>
<th>G</th>
<th>$C_G(t) = O_3(C_G(t)).H_t.K_t$</th>
<th>$H_t$</th>
<th>$D_3(G)^t$</th>
<th>$T$</th>
<th>$TC_G(T)$</th>
<th>$N_G(T)$</th>
<th>$D_3(G)^T$</th>
</tr>
</thead>
</table>
| $Fi'_{24}$ | $C(3A) = 3 \times O_8^+(3) : 3$  
$C(3C) = 3^7.2.U_4(3)$  
$C(3D) = 3^{2+4+6}(A_4 \times 2A_4)$  
$C(3E) = 3^2 \times G_2(3)$ | $O_8^+(3)$  
$G_2(3)$ | $D_4$  
point  
point  
point | $G_2$  
$G_2$  
$3^2$  
$3^2 \times G_2(3)$ | $(3^2 : 2 \times G_2(3)).2$  
$G_2$ |
Examples Involving Sporadic Simple Groups in Characteristic 3

<table>
<thead>
<tr>
<th>$G$</th>
<th>$C_G(t) = O_3(C_G(t)).H_t.K_t$</th>
<th>$H_t$</th>
<th>$D_3(G)^t$</th>
<th>$T$</th>
<th>$TC_G(T)$</th>
<th>$N_G(T)$</th>
<th>$D_3(G)^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Fi'_24$</td>
<td>$C(3A) = 3 \times O_8^+(3) : 3$</td>
<td>$O_8^+(3)$</td>
<td>$D_4$</td>
<td>$3^2$</td>
<td>$3^2 \times G_2(3)$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td></td>
<td>$C(3C) = 3^7.2.U_4(3)$</td>
<td></td>
<td>$G_2(3)$</td>
<td></td>
<td>$3^2 \times G_2(3)$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td></td>
<td>$C(3D) = 3^{2+4+6}.(A_4 \times 2A_4)$</td>
<td></td>
<td>$G_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C(3E) = 3^2 \times G_2(3)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

G2

Fall Central Sectional Meeting, University of Akron, 20-21 October 2012
### Examples Involving Sporadic Simple Groups in Characteristic 3

<table>
<thead>
<tr>
<th>G</th>
<th>$C_G(t) = O_3(C_G(t)).H_t.K_t$</th>
<th>$H_t$</th>
<th>$D_3(G)^t$</th>
<th>T</th>
<th>$TC_G(T)$</th>
<th>$N_G(T)$</th>
<th>$D_3(G)^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Fi'_24$</td>
<td>$C(3A) = 3 \times O_8^+(3) : 3$</td>
<td>$G_2(3)$</td>
<td>$D_4$ point</td>
<td>$3^2$</td>
<td>$3^2 \times G_2(3)$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td></td>
<td>$C(3C) = 3^7.2.\bar{U}_4(3)$</td>
<td>$G_2(3)$</td>
<td>$G_2$ point</td>
<td>$3^2$</td>
<td>$3^2 \times G_2(3)$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td></td>
<td>$C(3D) = 3^2+4+6.(A_4 \times 2A_4)$</td>
<td>$G_2(3)$</td>
<td>$3^2 \times G_2(3)$</td>
<td>$3^2 \times G_2(3)$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$(3^2 : 2 \times G_2(3)).2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td></td>
<td>$C(3E) = 3^2 \times G_2(3)$</td>
<td>$G_2(3)$</td>
<td>$G_2$ point</td>
<td>$3$</td>
<td>$3 \times G_2(3)$</td>
<td>$(3 \times G_2(3)) : 2$</td>
<td>$G_2$</td>
</tr>
<tr>
<td>$Th$</td>
<td>$C(3A) = 3 \times G_2(3)$</td>
<td>$G_2(3)$</td>
<td>$3 \times G_2(3)$</td>
<td>$3 \times G_2(3)$</td>
<td>$(3 \times G_2(3)) : 2$</td>
<td>$G_2$</td>
<td>$G_2$</td>
</tr>
</tbody>
</table>
### Examples Involving Sporadic Simple Groups in Characteristic 3

<table>
<thead>
<tr>
<th>G</th>
<th>$C_G(t) = O_3(C_G(t)).H_t.K_t$</th>
<th>$H_t$</th>
<th>$D_3(G)^t$</th>
<th>T</th>
<th>$T C_G(T)$</th>
<th>$N_G(T)$</th>
<th>$D_3(G)^T$</th>
</tr>
</thead>
</table>
| $Fi_24'$ | $C(3A) = 3 \times O_8^+(3) : 3$
$C(3C) = 3^{7.2}.U_4(3)$
$C(3D) = 3^{2+4+6}.(A_4 \times 2A_4)$
$C(3E) = 3^2 \times G_2(3)$ | $O_8^+(3)$
$G_2(3)$
$G_2(3)$
$G_2$ | $D_4$ point point $G_2$
$3^2$
$3^2$
$3^2 G_2(3)$
$(3^2 : 2 \times G_2(3)).2$
| $G_2$ | |
| $Th$ | $C(3A) = 3 \times G_2(3)$
$C(3C) = 3 \times 3^4 : 2A_6$ | $G_2(3)$
$G_2$
| $G_2$
| $3 \times G_2(3)$
$(3 \times G_2(3)) : 2$
| $G_2$ |

<table>
<thead>
<tr>
<th>G</th>
<th>$C_G(t) = O_3(C_G(t)).H_t.K_t$</th>
<th>$H_t$</th>
<th>$D_3(G)^t$</th>
<th>T</th>
<th>$T C_G(T)$</th>
<th>$N_G(T)$</th>
<th>$D_3(G)^T$</th>
</tr>
</thead>
</table>
| $M$   | $C(3A) = 3 \cdot Fi_24'$
$C(3C) = 3 \times Th$ | $Fi_24'$
$Th$
$D_3(Fi_24')$
$D_3(Th)$ | $3^{1+2}$
$3^2$
| $3^{1+2} \times G_2(3)$
$3^2 \times G_2(3)$
| $(3^{1+2} : 2^2 \times G_2(3)).2$
$(3^{1+2} : 2 \times G_2(3)).2$
| $G_2$
| $G_2$ |
Thank You
Thank You

The End