Practice Problems for Exam 1

Sections included: 1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4.

Table provided:

Rules of Inference

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1. Write each of these statements in the form “if p, then q” in English.
   a). It snows whenever the wind blows from the northeast.
   b). The apple trees will bloom if it stays warm for a week.
   c). That the Pistons win the championship implies that they beat the Lakers.
   d). It is necessary to walk 8 miles to get to the top of Long’s Peak.
   e). If you drive more than 400 miles, you will need to buy gasoline.
   f). Your guarantee is good only if you bought your iPod less than 90 days ago.

2. State the converse, contrapositive, and inverse of each of these implications.
   a). If it snows today, I will ski tomorrow.
   b). I come to class whenever there is going to be a quiz.
   c). A positive integer is a prime only if it has no divisors other than 1 and itself.

3. Show that \((p \rightarrow q) \land (q \rightarrow r)\) and \(p \rightarrow (q \land r)\) are logically equivalent.
4. Show that \((p \rightarrow r) \lor (q \rightarrow r)\) and \((p \land q) \rightarrow r\) are logically equivalent.
5. Show that \((p \lor q) \land (\sim p \lor r) \rightarrow (q \lor r)\) is a tautology.
6. Show that \((p \rightarrow q) \land (q \rightarrow r) \rightarrow (p \rightarrow r)\) is a tautology.

7. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?
   a). If \(n\) is a real number such that \(n > 1\), then \(n^2 > 1\). Suppose that \(n^2 > 1\). Then \(n > 1\).
   b). If \(n\) is a real number such that \(n > 2\), then \(n^2 > 4\). Suppose that \(n \leq 2\). Then \(n^2 \leq 4\).
   c). If \(n\) is a real number such that \(n > 3\), then \(n^2 > 9\). Suppose that \(n^2 \leq 9\). Then \(n \leq 3\).

8. Use truth tables to determine whether the following argument forms are valid:
   a). \[p \rightarrow q\] \[q \rightarrow p\] \[\therefore p \lor q\]
   b). \[p\] \[p \rightarrow q\] \[\sim q \lor r\] \[\therefore r\]
9. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantee its validity. Otherwise, state whether the converse or the inverse error is made.
   a). If I go to the movies, I won’t finish my homework.
      If I don’t finish my homework, I won’t do well on the exam tomorrow.
      Therefore, if I go to the movies, I won’t do well on the exam tomorrow.
   b). If this computer program is correct, then it produces the correct output when run with the test data my teacher gave me.
      This computer program produces the correct output when run with the test data my teacher gave me.
      Therefore this computer program is correct.

10. Express each of these statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers.
   a). The sum of two negative integers is negative.
   b). The difference of two positive integers is not necessarily positive.
   c). The sum of the squares of two integers is greater than or equal to the square of their sum.

11. Express the negations of each of these statements:
   a). $\exists x \forall y (x + y = y)$
   b). $\forall x \forall y (((x \geq 0) \land (y < 0)) \rightarrow (x - y > 0))$
   c). $\exists x \exists y (((x \leq 0) \land (y \leq 0)) \lor (x - y > 0))$
   d). $\exists x \exists y (x + 2y = 5 \land 2x + 4y = 2)$
   e). $\forall x \exists y ((x + 2y = 3) \lor (x = y) \rightarrow (x^2 = y^2))$
   f). $\forall x \exists y (x^2 = y^2 \rightarrow (x + 2y = 3) \land (x = y))$
   g). $\forall x ((\exists y (x = 2y)) \lor (\exists z (x = 2z + 1)))$

12. Let $P(m, n)$ be the statement “$m$ divides $n$” where the domain for both $n$ and $m$ is the set of positive integers. Determine the truth values of each of these statements:
   a). $P(2, 8)$
   b). $P(3, 5)$
   c). $\forall m \forall n P(m, n)$
   d). $\exists m \forall n P(m, n)$
   e). $\exists n \forall m P(m, n)$
   f). $\forall n P(1, n)$