1. There are 2500 computer science students at a school. Of these, 1700 have taken a course in Pascal, 1000 have taken a course in Fortran, and 350 have taken a course in C. Further, 800 have taken courses in both Pascal and Fortran, 250 have taken courses in both Fortran and C, and 300 have taken courses in both Pascal and C. If 200 of these students have taken courses in Fortran, Pascal, and C, how many of these 2500 students have not taken a course in any of these three programming languages?

\[
2500 - (1700 + 1000 + 350 - 800 - 250 - 300 + 200) = 600
\]

2. How many derangements of \{1, 2, 3, 4, 5, 6\} begin with the integers 1, 2 and 3, in some order?

The number of derangements of a set with 3 elements is \(D_3 = 2\).

Thus the answer is \(2 \times 2 = 4\).

3. In how many ways can you put 5 nonattacking rooks on the 5-by-5 chessboard with forbidden positions shown.

\[
\begin{array}{cccc}
X & X & & & \\
X & & & & \\
X & X & & & \\
& & & & X \\
& & & X & \\
\end{array}
\]

The coefficients are: \(r_1 = 7\), \(r_2 = 16\), \(r_3 = 13\), \(r_4 = 3\), \(r_5 = 0\).

Then we get \(5! - 7 \cdot 4! + 16 \cdot 3! - 13 \cdot 2! + 3 = 25\).
4. Find a closed form for the generating function for the sequence:

\[-1, -1, -1, -1, -1, -1, 0, 0, 0, \ldots\]

This is the polynomial:

\[-1 - x - x^2 - x^3 - x^4 - x^5 - x^6 = \frac{1 - x^7}{1 - x}\]

5. Solve the following nonhomogeneous recurrence relation:

\[h_n = 5h_{n-1} - 6h_{n-2} + 2n + 5, \quad (n \geq 2)\]

with initial values \(h_0 = 10, h_1 = 10\).

The homogeneous relation \(h_n - 5h_{n-1} + 6h_{n-2} = 0\) has general solution

\[h_n = c_1 \cdot 2^n + c_2 \cdot 3^n\]

For the particular solution, guess \(h_n = an + b\). This gives the following:

\[an + b = 5[a(n - 1) + b] - 6[a(n - 2) + b] + 2n + 5\]

and we get \(a = 1\) and \(b = 6\).

Therefore: \(h_n = c_1 \cdot 2^n + c_2 \cdot 3^n + n + 6\). Next we find \(c_1\) and \(c_2\):

\[10 = c_1 + c_2 + 6\]
\[10 = 2c_1 + 3c_2 + 7\]

which give \(c_1 = 9\) and \(c_2 = -5\). Therefore:

\[h_n = 9 \cdot 2^n - 5 \cdot 3^n + n + 6\]
6. Show that the sequence \( h_n = 3 \cdot (-1)^n + 2^n - n + 2 \) is a solution of the recurrence relation 
\[ h_n = h_{n-1} + 2h_{n-2} + 2n - 9 \]
You don’t need to solve the recurrence relation.

Substitute the formula \( h_n = 3 \cdot (-1)^n + 2^n - n + 2 \) in the right hand side of the recurrence relation and do the algebra:

\[
h_{n-1} + 2h_{n-2} + 2n - 9 = 3(-1)^{n-1} + 2^{n-1} - n + 1 + 2 + 6(-1)^{n-2} + 2 \cdot 2^{n-2} - 2n + 4 + 4 + 2n - 9
\]

which gives:

\[
h_{n-1} + 2h_{n-2} + 2n - 9 = 3 \cdot (-1)^{n} + 2^n - n + 2
\]

Thus the sequence \( h_n \) satisfies the given recurrence relation.

7. Solve the following recurrence relation by using the method of generating functions:

\[ h_n = 2h_{n-1}, \quad (n \geq 1); \quad h_0 = 2 \]

Rewrite the recurrence relation: \( h_n - 2h_{n-1} = 0 \).

Let \( g(x) = h_0 + h_1 x + h_2 x^2 + \ldots \)

and \(-2xg(x) = -2h_0 x - 2h_1 x^2 - \ldots \)

Add these two relations term by term and use the initial values to get:

\[
g(x) = \frac{2}{1 - 2x} = \sum_{n=0}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} 2^{n+1} x^n
\]

Therefore \( h_n = 2^{n+1} \).

8. Provide a closed formula for the sequence determined by the following generating function:

\[
\frac{x^2}{(1 - x)^3}
\]

\[
\frac{x^2}{(1 - x)^3} = x^2 \sum_{n=0}^{\infty} \binom{n + 2}{n} x^n = \sum_{n=0}^{\infty} \binom{n + 2}{2} x^{n+2} = \sum_{n=0}^{\infty} \binom{n}{2} x^n
\]

Therefore the sequence is:

\[
h_n = \binom{n}{2} = \frac{n(n - 1)}{2}
\]

9. Use generating functions to determine the number of different ways 12 identical action figures can be given to five children so that each child receives at least two action figures.

You must use generating functions for full credit.

\[
(x^2 + x^3 + \ldots)^5 = \frac{x^{10}}{(1 - x)^5} = x^{10} \sum_{n=0}^{\infty} \binom{n + 4}{4} x^n
\]

We need the coefficient of \( x^{12} \) in the above expression, which is:

\[
\binom{n + 2}{4} = \binom{6}{4} = 15
\]
10. In how many ways can eight distinct balls be distributed into three distinct urns if each urn must contain at least one ball?

If you use generating functions:

\[(x + x^2 + \ldots)^3 = x^3(1 + \ldots)^3 = \frac{x^3}{(1 - x)^3} = x^3 \sum_{n=0}^{\infty} \binom{n + 2}{2} x^n\]

then the answer is given by the coefficient of \(x^8\), which is

\[\binom{5 + 2}{2} = \binom{7}{2} = 21\]

Or you may use methods from previous sections on combinations with repetitions. Start with:

\[e_1 + e_2 + e_3 = 8\]

with \(e_i \geq 2\) for \(i = 1, 2, 3\). Let \(y_i = e_i - 1\) and rewrite the equation as:

\[y_1 + y_2 + y_3 = 5\]

Then the answer is given by:

\[\binom{5 + 3 - 1}{5} = \binom{7}{5} = 21\]