Math 46 - Section 020
Final Exam
June 13, 2006

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Instructions

- Turn off all electronic devices NOW. If you do not do so, then you will not receive any credit for your exam.
- There are 11 pages on this exam with 10 problems. Before you begin, make sure that your exam has all 10 pages.
- The examination period is from 3:00 pm to 6:00 pm. If you wish to receive credit for your exam, then make sure that your exam is submitted for grading by 6:00 pm.
- You may not use a calculator during the exam.
- You may refer to any material on your 8½ × 11 inch sheet of paper that you brought with you; you may not use any other reference.
- To receive full credit for a problem, you must provide a correct answer and a sufficient amount of work so that it can be determined how you arrived at your answer.
- Clearly indicate what your solutions are and any work that you do not want to be included in the grading process.
- If you wish to speak with the proctor during the exam, then raise your hand and the proctor will come to you.
- If it is determined that you have given or received any unauthorized aid during the exam, then you will receive no credit for your exam.
- Each problem is 14 points worth (140 points total).
1. Use the principle of superposition to find a particular solution; then find the general solution of the following differential equation: 
   \[ y'' - 4y' + 4y = 2 + 8x - 4x^2 + e^x. \]

2. Find the general solution of the differential equation: 
   \[ x^2y'' - 3xy' + 4y = 4x^4, \]
   given that \( y_1 = x^2 \) is a solution of the complementary equation. You must use reduction of order.

3. Find a differential equation whose solution is:
   \[ y(x) = \left( x + \frac{1}{2} \right)e^x + C_1e^{2x} + C_2e^{-x}, \]
   Justify your answer.

4. Solve the initial value problem:
   \[ x^2 \frac{dy}{dx} + 2xy - 1 = 0, \quad y(1) = 2 \]

5. Find a particular solution for the following differential equation:
   \[ x^2y'' - 4xy' + 6y = \frac{x^5}{1 + x^2} \]
   given that \( y_1 = x^2 \) and \( y_2 = x^3 \) is a fundamental set of solutions for the complementary equation.

6. A mass of 1 kg is attached to a spring with a spring constant of 3 kg/sec\(^2\). The spring has a damping constant of 4 kg/sec. The spring is under an external force of 65 cos 2t N. Find the steady state component of the displacement and the displacement if the initial conditions are \( y(0) = 0 \) m and \( y'(0) = 17 \) m/sec.

7. Solve the :
   \[ \frac{dy}{dx} = \frac{3x^2 - 1}{3 + 2y} \]
   Write the solution in explicit form.

8. Find an integrating factor that is a function of only one variable and solve the given equation:
   \[ (x^4y^3 + y)dx + (x^5y^2 - x)dy = 0 \]
   Leave your answer in implicit form.

9. A mass of 1 kg is attached to a spring causing it to stretch .1 m. The spring is then placed in free motion. If you don’t know the damping constant, what is the range of possible values for the quasiperiod (the time for the spring to complete one cycle)? Use \( g = 9.8 \) m/sec\(^2\).

10. Use Laplace transforms to solve the following initial value problem:
    \[ y'' + 4y' + 3y = 1, \quad y(0) = 0, \quad y'(0) = 0 \]