**Homework 1**

Math 2568 Due: January 16, 2019

In Exercises 1 – 3, let $x = (2,1,3)$ and $y = (1,1,-1)$ and compute the given expression.

**§1.1, Exercise 2.** $2x - 3y$.

$2x - 3y = (4,2,6) - (3,3,-3) = (1,-1,9)$.

**§1.1, Exercise 4.** Let $A$ be the $3 \times 4$ matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 3 & 4 & -7 & 10 \\ 6 & -3 & 4 & 2 \end{pmatrix}.$$

(a) For which $n$ is a row of $A$ a vector in $\mathbb{R}^n$? .

(b) What is the $2^{nd}$ column of $A$?

(c) Let $a_{ij}$ be the entry of $A$ in the $i^{th}$ row and the $j^{th}$ column. What is $a_{23} - a_{31}$?

(a) The number of entries in a row is the number of columns. Thus, $n = 4$;
(b) $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$; (c) $a_{23} - a_{31} = -7 - 6 = -13$.

For each of the pairs of vectors or matrices in Exercises 5 – 9, decide whether addition of the members of the pair is possible; and, if addition is possible, perform the addition.

**§1.1, Exercise 7.** $x = (1,2,3)$ and $y = (-2,1)$.

$x$ has three entries; $y$ has two entries; addition is not possible.

In Exercises 10 – 11, let $A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$ and compute the given expression.

**§1.1, Exercise 10.** $4A + B$.

$4A + B = \begin{pmatrix} 8 & 6 \\ -1 & 15 \end{pmatrix}$.

In Exercises 3 – 4, let $x = (1.2, 1.4, -2.45)$ and $y = (-2.6, 1.1, 0.65)$ and use MATLAB to compute the given expression.

**§1.2, Exercise 3.** $3.27x - 7.4y$.

$3.27x - 7.4y = (23.1640, -3.5620, -12.8215)$.

In Exercises 5 – 6, let

$$A = \begin{pmatrix} 1.2 & 2.3 & -0.5 \\ 0.7 & -1.4 & 2.3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2.9 & 1.23 & 1.6 \\ -2.2 & 1.67 & 0 \end{pmatrix}$$

and use MATLAB to compute the given expression.

**§1.2, Exercise 5.** $-4.2A + 3.1B$.

$-4.2A + 3.1B = \begin{pmatrix} -14.0300 & -5.8470 & 7.0600 \\ -9.7600 & 11.0570 & -9.6600 \end{pmatrix}$.

In Exercises 1 – 5 decide whether or not the given matrix is symmetric.

**§1.3, Exercise 5.** $A = \begin{pmatrix} 3 & 4 & -1 \\ 4 & 3 & 1 \\ -1 & 1 & 10 \end{pmatrix}$.

Since $a_{21} = a_{12}$, $a_{31} = a_{13}$, and $a_{32} = a_{23}$, the matrix is symmetric.

A general $2 \times 2$ diagonal matrix has the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Thus the two unknown real numbers $a$ and $b$ are needed to specify each $2 \times 2$ diagonal matrix. In Exercises 11 – 16, how many unknown real numbers are needed to specify each of the given matrices:

**§1.3, Exercise 11.** An upper triangular $2 \times 2$ matrix?

A $2 \times 2$ upper triangular matrix $A$ has the form $A = \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}$. Thus the number of entries needed to define $A$ is 3.

**§1.3, Exercise 13.** An $m \times n$ matrix?

Each row of the matrix has $n$ entries and there are $m$ rows. Hence the number of unknown entries is $mn$.

**§1.3, Exercise 16.** A symmetric $n \times n$ matrix?

The number of independent entries in row $k$ of an $n \times n$ symmetric matrix is $n - k + 1$. Thus the number of independent entries in the matrix is

$$n + (n-1) + \cdots + 1 = 1 + 2 + \cdots + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

In each of Exercises 17 – 19 determine whether the statement is *True* or *False*?

**§1.3, Exercise 18.** Every diagonal matrix is a multiple of the identity matrix.

$False$ — for example: $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

**§1.4, Exercise 9.** Find a real number $a$ so that the vectors

$$x = (1, 3, 2) \quad \text{and} \quad y = (2, a, -6)$$

are perpendicular.

The vectors $x$ and $y$ are perpendicular when $(1, 3, 2) \cdot (2, a, -6) = 3a - 10 = 0$. Thus, $a = \frac{10}{3}$.

In Exercises 21– 23 find the angle in degrees between the given pair of vectors.

**§1.4, Exercise 21.** $x = (2, 1, -3, 4)$ and $y = (1, 1, -5, 7)$.

$\theta = \arccos\left(\frac{x \cdot y}{||x|| \, ||y||}\right) = 0.2715 \text{ radians} = 15.5570^\circ$.