Homework 2

Math 2568 Due: Wednesday, January 23, 2019 at beginning of class

Problem 1

Determine whether the given pair of vectors is perpendicular.

§1.4, Exercise 8. $x = (2, 1, 4, 5)$ and $y = (1, -4, 3, -2)$.

Answer: The vectors are perpendicular.

Solution: Compute: $(2, 1, 4, 5) \cdot (1, -4, 3, -2) = 0$.

Problem 2 (MATLAB)

§2.1, Exercise 14.(MATLAB) Suppose that the four substances S_1, S_2, S_3, S_4 contain the following percentages of vitamins A, B, C and F by weight

Vitamin	S_1	S_2	S_3	S_4
A	25%	19%	20%	3%
B	2%	14%	2%	14%
C	8%	4%	1%	0%
F	25%	31%	25%	16%

Mix the substances S_1, S_2, S_3 and S_4 so that the resulting mixture contains precisely 3.85 grams of vitamin A, 2.30 grams of vitamin B, 0.80 grams of vitamin C, and 5.95 grams of vitamin F. How many grams of each substance have to be contained in the mixture?

Discuss what happens if we require that the resulting mixture contains 2.00 grams of vitamin B instead of 2.30 grams.

Answer: The vector of solutions is:

```
ans =
    7.3828
    4.1016
    4.5313
   10.6250
```

Solution: First, translate the data in the table to a system of linear equations, relating the quantities of S_1, S_2, S_3, and S_4 in the mixture to the quantities of vitamins A, B, C, and F. The first equation is $.25S_1 + .19S_2 + .20S_3 + .03S_4 = A$, and the other three equations correspond to the other vitamins. From this data, find the coefficient matrix A for the system. The desired quantities of each vitamin form the solution vector b.

```
A =                                                        b =
    0.2500    0.1900    0.2000    0.0300                      3.8500
    0.0200    0.1400    0.0200    0.1400                      2.3000
    0.0800    0.0400    0.0100         0                      0.8000
    0.2500    0.3100    0.2500    0.1600                      5.9500
```

As in the previous problems, the system can be solved by typing `A\b`.

If $b(2) = 2.00$ instead of 2.30, then `A\b` yields

```
ans =
   22.4023
  -27.8320
   12.1094
   37.1875
```

Note that the components of the answer vector refer to weights of substances, which cannot be negative. This answer contains a negative component; so although a mathematically valid solution exists, we cannot mix the substances in such a way that

$$b = \begin{pmatrix} 3.85 \\ 2.00 \\ 0.80 \\ 5.95 \end{pmatrix}.$$

Problem 3

§2.2, Exercise 5.

(a) Find a vector u normal to the plane $2x + 2y + z = 3$.

(b) Find a vector v normal to the plane $x + y + 2z = 4$.

(c) Find the cosine of the angle θ between the vectors u and v.

(a) $u = (2, 2, 1)$, since we know that the normal vector to the plane $ax+by+cz = d$ is (a, b, c).

(b) $v = (1, 1, 2)$.

(c) $\cos\theta = \frac{u \cdot v}{||u||\ ||v||} = \frac{2}{\sqrt{6}}$. In MATLAB, type `acos(2/sqrt(6))*180/pi` to obtain $\theta = 35.2644°$.

Problem 4 (MATLAB)

§2.2, Exercise 10.(MATLAB) Use MATLAB to determine graphically the geometry of the set of solutions to the system of equations:

$$\begin{array}{rcrcrcl} x & + & 3y & + & 4z & = & 5 \\ 2x & + & y & + & z & = & 1 \\ -4x & + & 3y & + & 5z & = & 7. \end{array}$$

Attempt to use MATLAB to find an exact solution to this system and discuss the implications of your calculations.

Hint: After setting up the graphics display in MATLAB, you can use the command `view([0,1,0])` to get a better view of the solution point.

Answer: The solution set is a line because the three planes intersect in a line.

Solution: If the left-hand side of the system is entered into MATLAB as matrix `A`, and the solution vector is entered as `b`, then typing `A\b` yields

```
Warning: Matrix is singular to working precision.
ans =
   Inf
   Inf
   Inf
```

Problem 5

Determine the augmented matrix and all solutions for each system of linear equations

§2.3, Exercise 11. $\begin{array}{rcl} 2x - y + z + w & = & 1 \\ x + 2y - z + w & = & 7 \end{array}.$

The augmented matrix for this system is

$$\left(\begin{array}{rrrr|r} 2 & -1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 1 & 7 \end{array}\right)$$

which can be row reduced to

$$\left(\begin{array}{rrrr|r} 1 & 0 & \frac{1}{5} & \frac{3}{5} & \frac{9}{5} \\ 0 & 1 & -\frac{3}{5} & \frac{1}{5} & \frac{13}{5} \end{array}\right).$$

The solution set is therefore

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{9}{5} - \frac{1}{5}x_3 - \frac{3}{5}x_4 \\ \frac{13}{5} + \frac{3}{5}x_3 - \frac{1}{5}x_4 \\ x_3 \\ x_4 \end{pmatrix}.$$

Problem 6

Consider the augmented matrices representing systems of linear equations, and decide

(a) if there are zero, one or infinitely many solutions, and

(b) if solutions are not unique, how many variables can be assigned arbitrary values.

§2.3, Exercise 14. $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 5 & 0 & 2 \\ 0 & 0 & 4 & 3 \end{array}\right)$.

Answer: The system has a unique solution.

Solution: The row-reduced form of the matrix is:

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{3}{4} \end{array}\right).$$

Problem 7 (MATLAB)

Use elementary row operations and MATLAB to put each of the given matrices into row echelon form. Suppose that the matrix is the augmented matrix for a system of linear equations. Is the system consistent or inconsistent?

§2.3, Exercise 24.(MATLAB)

$$\begin{pmatrix} -2 & 1 & 9 & 1 \\ 3 & 3 & -4 & 2 \\ 1 & 4 & 5 & 5 \end{pmatrix}.$$

The row-reduced matrix is:

```
A =
    1.0000   -0.5000   -4.5000   -0.5000
         0    1.0000    2.1111    0.7778
         0         0         0    2.0000
```

This matrix represents an inconsistent linear system.

Problem 8

§2.4, Exercise 3. The augmented matrix of a consistent system of five equations in seven unknowns has rank equal to three. How many parameters are needed to specify all solutions?

Answer: Four parameters are needed to specify all solutions.

Solution: According to Theorem 2.4.6, $n - \ell$ parameters are needed to parameterize the set of all solutions of a linear system, where n is the number of unknowns, and ℓ is the rank of the reduced echelon matrix. In this case, $n = 7$ and $\ell = 3$.

Problem 9 (MATLAB)

Compute the rank of the given matrix.

§2.4, Exercise 10.(MATLAB) $\begin{pmatrix} 2 & 1 & 0 & 1 \\ -1 & 3 & 2 & 4 \\ 5 & -1 & 2 & -2 \end{pmatrix}$.

The rank of the matrix is 3.