Homework 4
Math 2568

Problem 1

§3.4, Exercise 2. Write all solutions to the homogeneous system of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_4 - x_5 &= 0 \\ x_3 - 2x_4 + x_5 &= 0 \end{aligned}$$

as the general superposition of three vectors.

Problem 2

§3.4, Exercise 3.

(a) Find all solutions to the homogeneous equation $Ax = 0$ where

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix}.$$

(b) Find a single solution to the inhomogeneous equation

$$Ax = \begin{pmatrix} 6 \\ 6 \end{pmatrix}. \tag{1}$$

(c) Use your answers in (a) and (b) to find all solutions to (1).

Problem 3

§3.5, Exercise 11. Let

$$A = \begin{pmatrix} 1 & 0 & -3 \\ -2 & 1 & 1 \\ 0 & 1 & -5 \end{pmatrix}.$$

Let A^t is the transpose of the matrix A, as defined in Section 1.3. Compute AA^t.

Problem 4

§3.5, Exercise 10. Let

$$A = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} a & 3 \\ b & 2 \end{pmatrix}.$$

For which values of a and b does $AB = BA$?

Problem 5 (MATLAB)

Decide for the given pair of matrices A and B whether or not the products AB or BA are defined and compute the products when possible.

§3.5, Exercise 14.(MATLAB)

$$A = \begin{pmatrix} -2 & -2 & 4 & 5 \\ 0 & -3 & -4 & 3 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & 0 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 3 & -4 & 5 \\ 4 & -3 & 0 & -2 \\ -3 & -4 & -4 & -3 \\ -2 & -2 & 3 & -1 \end{pmatrix} \tag{2}$$

Problem 6

§3.6, Exercise 4. Let

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

(a) Show that $J^2 = -I$.

(b) Evaluate $(aI + bJ)(cI + dJ)$ in terms of I and J.

Problem 7 (MATLAB)

§3.6, Exercise 9.(MATLAB) Use the `rand(3,3)` command in MATLAB to choose five pairs of 3×3 matrices A and B at random. Compute AB and BA using MATLAB to see that in general these matrix products are unequal.

Problem 8

Use row reduction to find the inverse of the given matrix.

§3.7, Exercise 5. $\begin{pmatrix} 1 & 4 & 5 \\ 0 & 1 & -1 \\ -2 & 0 & -8 \end{pmatrix}$.

Problem 9 (MATLAB)

§3.7, Exercise 12.(MATLAB) Try to compute the inverse of the matrix

$$C = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 0 & 2 & 1 \end{pmatrix} \tag{3}$$

in MATLAB using the command `inv`. What happens — can you explain the outcome?

Now compute the inverse of the matrix

$$\begin{pmatrix} 1 & \epsilon & 3 \\ -1 & 2 & -2 \\ 0 & 2 & 1 \end{pmatrix}$$

for some nonzero numbers ϵ of your choice. What can be observed in the inverse if ϵ is very small? What happens when ϵ tends to zero?

Problem 10

§3.7, Exercise 14. True or False: Determine whether the following statements are true or false, and explain your answer.

(a) The only 3×2 matrix A so that $Ax = 0$ for all $x \in \mathbb{R}^2$ is $A = 0$.

(b) A system of 5 equations in 3 unknowns with the solution $x_1 = 0, x_2 = -3, x_3 = 1$ must have infinitely many solutions.

(c) If A is a 2×2 matrix and $A^2 = 0$, then $A = 0$.

(d) If $u, v \in \mathbb{R}^3$ are perpendicular, then $\|u + v\| = \|u\| + \|v\|$.