Homework 7
Math 2568 Mar 20, 2019

Problem 1

You are given a pair of vectors v_1, v_2 spanning a subspace of $\mathbb{R}^3$. Decide whether that subspace is a line or a plane through the origin. If it is a plane, then compute a vector N that is perpendicular to that plane.

§5.6, Exercise 3. $v_1 = (0, 1, 0) \quad \text{and} \quad v_2 = (4, 1, 0)$.

Problem 2

§5.6, Exercise 4. The pairs of vectors

$$v_1 = (-1, 1, 0) \quad \text{and} \quad v_2 = (1, 0, 1)$$

span a plane P in $\mathbb{R}^3$. The pairs of vectors

$$w_1 = (0, 1, 0) \quad \text{and} \quad w_2 = (1, 1, 0)$$

span a plane Q in $\mathbb{R}^3$. Show that P and Q are different and compute the subspace of $\mathbb{R}^3$ that is given by the intersection $P \cap Q$.

Problem 3

§5.6, Exercise 6. Let

$$A = \begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 1 & 5 & 7 \\ 3 & 4 & 4 & 11 \end{pmatrix}.$$

(a) Find a basis for the subspace $\mathcal{C} \subset \mathbb{R}^3$ spanned by the columns of A.

(b) Find a basis for the subspace $\mathcal{R} \subset \mathbb{R}^4$ spanned by the rows of A.

(c) What is the relationship between $\dim \mathcal{C}$ and $\dim \mathcal{R}$?

Problem 4

§5.6, Exercise 8. Let W be an infinite dimensional subspace of the vector space V. Show that V is infinite dimensional.

Problem 5

§5.6, Exercise 13. Let A be an $m \times n$ matrix and B be an $n \times k$ matrix.

(a) Show that null space$(B) \subseteq$ null space(AB).

(b) Show that nullity$(B) \leq$ nullity(AB).

Problem 6

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 15. Every set of three vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$.

Problem 7

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 17. If $\{v_1, v_2\}$ is a basis for the plane $z = 0$ in $\mathbb{R}^3$, then $\{v_1, v_2, e_3\}$ is a basis for $\mathbb{R}^3$.

Problem 8

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 18. If $\{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$, the only subspaces of R^3 of dimension one are $\text{span}\{v_1\}, \text{span}\{v_2\}$, and $\text{span}\{v_3\}$.

Problem 9

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 19. The only subspace of $\mathbb{R}^3$ that contains finitely many vectors is $\{0\}$.

Problem 10

Compute the general solution for the given system of differential equations.

§6.2, Exercise 4. $\dfrac{dX}{dt} = \begin{pmatrix} -1 & -4 \\ 2 & 3 \end{pmatrix} X.$

Problem 11

Compute the general solution for the given system of differential equations.

§6.2, Exercise 5. $\dfrac{dX}{dt} = \begin{pmatrix} 8 & -15 \\ 3 & -4 \end{pmatrix} X.$