

**Problem 1.** True or false? Explain your reasoning.

- (1) If  $A$  and  $B$  are matrices such that  $AB = I_n$ , then  $BA = I_n$ .
- (2) If  $A, B, C$  are matrices such that  $AB = I_n$  and  $BC = I_n$ , then  $A = C$ .
- (3) If the system of equations  $Ax = b$  has a unique solution, then  $A$  is invertible.
- (4) If  $L : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  is a linear transformation and  $A$  is a  $3 \times 3$  matrix, then the map  $x \mapsto A(Lx)$  is linear.
- (5) If  $A$  is an  $m \times n$  matrix and  $Ax = b$  is inconsistent, then  $m > n$ .

**Problem 2.** Find all solutions to the system of linear equations

$$\begin{aligned}x_1 + 2x_2 + 2x_4 + x_5 &= 2 \\x_1 + 2x_3 + x_5 &= -2 \\x_2 - x_3 + x_4 &= 2\end{aligned}$$

**Problem 3.** Suppose  $L : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  is a linear transformation such that

$$L \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad L \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

Find the  $3 \times 2$  matrix  $A$  such that  $L = L_A$ .

**Problem 4.** Determine whether the following matrices are invertible. If they are, find their inverses. Explain all your reasoning.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$$

**Problem 5.** Consider the system of equations

$$\begin{aligned}x_1 + 3x_3 &= 1 \\-x_1 + 2x_2 - 3x_3 &= 1 \\2x_2 + ax_3 &= b.\end{aligned}$$

For which values of  $a, b \in \mathbb{R}$  does this system have no solutions, a unique solution, or infinitely many solutions? Your answer should split the possibilities for  $a, b \in \mathbb{R}$  into 3 disjoint sets.