**Math 2568 Homework 10**

Math 2568 Due: Wednesday, November 13, 2019

# Problem 1

**§6.3, Exercise 2.** Use (4.6.13) in Chapter 3 to verify that the traces of similar matrices are equal.

# Problem 2

Determine whether or not the given matrices are similar, and why.

**§6.3, Exercise 3.** $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ -3 & 8 \end{pmatrix}$.

# Problem 3

**§6.3, Exercise 5.** Let $B = P^{-1}AP$ so that $A$ and $B$ are similar matrices. Suppose that $v$ is an eigenvector of $B$ with eigenvalue $\lambda$. Show that $Pv$ is an eigenvector of $A$ with eigenvalue $\lambda$.

# Problem 4

Determine whether or not the equilibrium at the origin in the system of differential equations $\dot{X} = CX$ is asymptotically stable.

**§6.4, Exercise 1.** $C = \begin{pmatrix} 1 & 2 \\ 4 & 1 \end{pmatrix}$.

# Problem 5

Determine whether the equilibrium at the origin in the system of differential equations $\dot{X} = CX$ is a sink, a saddle or a source.

**§6.4, Exercise 5.** $C = \begin{pmatrix} 3 & 5 \\ 0 & -2 \end{pmatrix}$.

# Problem 6

compute the determinants of the given matrix.

**§7.1, Exercise 1.** $A = \begin{pmatrix} -2 & 1 & 0 \\ 4 & 5 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

# Problem 7

compute the determinants of the given matrix.

**§7.1, Exercise 3.** $C = \begin{pmatrix} 2 & 1 & -1 & 0 & 0 \\ 1 & -2 & 3 & 0 & 0 \\ -3 & 2 & -2 & 0 & 0 \\ 1 & 1 & -1 & 2 & 4 \\ 0 & 2 & 3 & -1 & -3 \end{pmatrix}$.

# Problem 8

Use row reduction to compute the determinant of the given matrix.

**§7.1, Exercise 6.** $A = \begin{pmatrix} -1 & -2 & 1 \\ 3 & 1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$.

# Problem 9

Determine the characteristic polynomial and the eigenvalues of the given matrices.

**§7.2, Exercise 2.** $B = \begin{pmatrix} 2 & 1 & -5 & 2 \\ 1 & 2 & 13 & 2 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$.

## Problem 10

**§7.2, Exercise 3.** Find a basis for the eigenspace of

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

corresponding to the eigenvalue $\lambda = 2$.

## Problem 11

**§7.2, Exercise 4.** Consider the matrix

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(a) Verify that the characteristic polynomial of $A$ is $p_\lambda(A) = (\lambda - 1)(\lambda + 2)^2$.

(b) Show that $(1, 1, 1)$ is an eigenvector of $A$ corresponding to $\lambda = 1$.

(c) Show that $(1, 1, 1)$ is orthogonal to every eigenvector of $A$ corresponding to the eigenvalue $\lambda = -2$.

## Problem 12

**§7.2, Exercise 5.** Let

$$A = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$$

One of the eigenvalues of $A$ is $-1$. Find the other eigenvalues of $A$.

## Problem 13

**§7.2, Exercise 7.** Find the characteristic polynomial and the eigenvalues of

$$A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}.$$

Find eigenvectors corresponding to each of the three eigenvalues.