Math 2568 Homework 11

Math 2568 Due: Monday, November 25, 2019

Problem 1

§7.2, Exercise 6. Consider the matrix $A = \begin{pmatrix} 8 & 5 \\ -10 & -7 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of A.

(b) Show that the eigenvectors found in (a) form a basis for $\mathbb{R}^2$.

(c) Find the coordinates of the vector (x_1, x_2) relative to the basis in part (b).

Problem 2

§7.2, Exercise 8. Let A be an $n \times n$ matrix. Suppose that

$$A^2 + A + I_n = 0.$$

Prove that A is invertible.

Problem 3

§7.2, Exercise 12. When n is odd show that every real $n \times n$ matrix has a real eigenvalue.

Problem 4

§7.3, Exercise 2. The eigenvalues of

$$A = \begin{pmatrix} -1 & 2 & -1 \\ 3 & 0 & 1 \\ -3 & -2 & -3 \end{pmatrix}$$

are $2, -2, -4$. Find the eigenvectors of A for each of these eigenvalues and find a 3×3 invertible matrix S so that $S^{-1}AS$ is diagonal.

Problem 5

§7.3, Exercise 3. Let

$$A = \begin{pmatrix} -1 & 4 & -2 \\ 0 & 3 & -2 \\ 0 & 4 & -3 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of A, and find an invertible matrix S so that $S^{-1}AS$ is diagonal.

Problem 6

§7.3, Exercise 4. Let A and B be similar $n \times n$ matrices.

(a) Show that if A is invertible, then B is invertible.

(b) Show that $A + A^{-1}$ is similar to $B + B^{-1}$.

Problem 7

§7.3, Exercise 6. Let A be an $n \times n$ real diagonalizable matrix. Show that $A + \alpha I_n$ is also real diagonalizable.

Problem 8

§7.3, Exercise 9. Let A be an $n \times n$ matrix all of whose eigenvalues equal ± 1. Show that if A is diagonalizable, the $A^2 = I_n$.

Problem 9

§8.1, Exercise 1. Use Theorem 8.1.2 and (8.1.3) to construct matrix of a linear mapping L from $\mathbb{R}^3$ to $\mathbb{R}^2$ with $L(v_i) = w_i$, $i = 1, 2, 3$, where

$$v_1 = (1, 0, 2) \quad v_2 = (2, -1, 1) \quad v_3 = (-2, 1, 0)$$

and

$$w_1 = (-1, 0) \quad w_2 = (0, 1) \quad w_3 = (3, 1).$$

Problem 10

§8.1, Exercise 2. Let $\mathcal{P}_n$ be the vector space of polynomials $p(t)$ of degree less than or equal to n. Show that $\{1, t, t^2, \ldots, t^n\}$ is a basis for $\mathcal{P}_n$.

Problem 11

§8.1, Exercise 3. Show that

$$\frac{d}{dt} : \mathcal{P}_3 \to \mathcal{P}_2$$

is a linear mapping.

Problem 12

§8.1, Exercise 4. Show that

$$L(p) = \int_0^t p(s) ds$$

is a linear mapping of $\mathcal{P}_2 \to \mathcal{P}_3$.