Math 2568 Homework 12

Math 2568 Due: Monday, December 4, 2019

Problem 1

§8.2, Exercise 3. Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 2 \\ 1 & 2 & -1 & 3 \end{pmatrix}.$$

(a) Find a basis for the row space of A and the row rank of A.

(b) Find a basis for the column space of A and the column rank of A.

(c) Find a basis for the null space of A and the nullity of A.

(d) Find a basis for the null space of A^t and the nullity of A^t.

Problem 2

§8.3, Exercise 2. Let $w_1 = (1, 2)$ and $w_2 = (0, 1)$ be a basis for $\mathbb{R}^2$. Let $L_A : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by the matrix

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

in standard coordinates. Find the matrix $[L]_{\mathcal{W}}$.

Problem 3

§8.3, Exercise 3. Let E_{ij} be the 2×3 matrix whose entry in the i^{th} row and j^{th} column is 1 and all of whose other entries are 0.

(a) Show that

$$\mathcal{V} = \{E_{11}, E_{12}, E_{13}, E_{21}, E_{22}, E_{23}\}$$

is a basis for the vector space of 2×3 matrices.

(b) Compute $[A]_{\mathcal{V}}$ where

$$A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & -2 & 4 \end{pmatrix}.$$

Problem 4

§8.3, Exercise 4. Verify that $\mathcal{V} = \{p_1, p_2, p_3\}$ where

$$p_1(t) = 1 + 2t, \quad p_2(t) = t + 2t^2, \quad \text{and} \quad p_3(t) = 2 - t^2,$$

is a basis for the vector space of polynomials $\mathcal{P}_2$. Let $p(t) = t$ and find $[p]_{\mathcal{V}}$.

Problem 5

§9.1, Exercise 1. Find an orthonormal basis for the solutions to the linear equation

$$2x_1 - x_2 + x_3 = 0.$$

Problem 6

§9.1, Exercise 2.

(a) Find the coordinates of the vector $v = (1, 4)$ in the orthonormal basis $\mathcal{V}$

$$v_1 = \frac{1}{\sqrt{5}}(1, 2) \quad \text{and} \quad v_2 = \frac{1}{\sqrt{5}}(2, -1).$$

(b) Let $A = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}$. Find $[A]_{\mathcal{V}}$.

Problem 7

§9.4, Exercise 1. Let

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

be the general real 2×2 symmetric matrix.

(a) Prove directly using the discriminant of the characteristic polynomial that A has real eigenvalues.

(b) Show that A has equal eigenvalues only if A is a scalar multiple of I_2.

Problem 8

§9.4, Exercise 2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of A and verify that the eigenvectors are orthogonal.

Problem 9

Decide whether or not the given matrix is orthogonal.

§9.4, Exercise 5. $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}.$

Problem 10

Decide whether or not the given matrix is orthogonal.

§9.4, Exercise 6. $\begin{pmatrix} \cos(1) & -\sin(1) \\ \sin(1) & \cos(1) \end{pmatrix}.$

Problem 11

Decide whether or not the given matrix is orthogonal.

§9.4, Exercise 7. $\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix}.$

Problem 12

§9.4, Exercise 8. Let Q be an orthogonal $n \times n$ matrix. Show that Q preserves the length of vectors, that is

$$\|Qv\| = \|v\| \quad \text{for all } v \in \mathbb{R}^n.$$