Math 2568 Homework 6

Math 2568 Due: Monday, October 7, 2019

Problem 1

You are given a vector space V and a subset W. For each pair, decide whether or not W is a subspace of V, and explain why.

§5.1, Exercise 6. $V = \mathbb{R}^2$ and W consists of vectors in $\mathbb{R}^2$ for which the sum of the components is 1.

Problem 2

You are given a vector space V and a subset W. For each pair, decide whether or not W is a subspace of V, and explain why.

§5.1, Exercise 8. $V = \mathcal{C}^1$ and W consists of functions $x(t) \in \mathcal{C}^1$ satisfying $\int_{-2}^{4} x(t)dt = 0$.

Problem 3

§5.1, Exercise 16. Let V be a vector space and let W_1 and W_2 be subspaces. Show that the intersection $W_1 \cap W_2$ is also a subspace of V.

Problem 4

§5.1, Exercise 18. For which scalars a, b, c, d do the solutions to the equation

$$ax + by + cz = d$$

form a subspace of $\mathbb{R}^3$?

Problem 5

A single equation in three variables is given. For each equation write the subspace of solutions in $\mathbb{R}^3$ as the span of two vectors in $\mathbb{R}^3$.

§5.2, Exercise 2. $x - y + 3z = 0$.

Problem 6

Each of the given matrices is in reduced echelon form. Write solutions of the corresponding homogeneous system of linear equations as a span of vectors.

§5.2, Exercise 8. $B = \begin{pmatrix} 1 & -1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$.

Problem 7

§5.2, Exercise 9. Write a system of two linear equations of the form $Ax = 0$ where A is a 2×4 matrix whose subspace of solutions in $\mathbb{R}^4$ is the span of the two vectors

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

Problem 8

§5.2, Exercise 20. Let $W \subset \mathbb{R}^4$ be the subspace that is spanned by the vectors

$$w_1 = (-1, 2, 1, 5) \quad \text{and} \quad w_2 = (2, 1, 3, 0).$$

Find a linear system of two equations such that $W = \text{span}\{w_1, w_2\}$ is the set of solutions of this system.

Problem 9

§5.2, Exercise 22. Let V be a vector space and let $v, w \in V$ be vectors. Show that

$$\text{span}\{v, w\} = \text{span}\{v, w, v + 3w\}.$$

Problem 10

§5.2, Exercise 24. Let $Ax = b$ be a system of m linear equations in n unknowns, and let $r = \text{rank}(A)$ and $s = \text{rank}(A|b)$. Suppose that this system has a unique solution. What can you say about the relative magnitudes of m, n, r, s?

Problem 11

§5.4, Exercise 1. Let w be a vector in the vector space V. Show that the sets of vectors $\{w, 0\}$ and $\{w, -w\}$ are linearly dependent.

Problem 12

§5.4, Exercise 3. Let

$$u_1 = (1, -1, 1) \quad u_2 = (2, 1, -2) \quad u_3 = (10, 2, -6).$$

Is the set $\{u_1, u_2, u_3\}$ linearly dependent or linearly independent?

Problem 13

§5.4, Exercise 8. Suppose that the three vectors $u_1, u_2, u_3 \in \mathbb{R}^n$ are linearly independent. Show that the set

$$\{u_1 + u_2, u_2 + u_3, u_3 + u_1\}$$

is also linearly independent.