Math 2568 Homework 7

Math 2568 Due: Monday, October 14, 2019

Problem 1

§5.4, Exercise 7. Show that the functions $f_1(t) = \sin t$, $f_2(t) = \cos t$, and $f_3(t) = \cos\left(t + \frac{\pi}{3}\right)$ are linearly dependent vectors in $\mathcal{C}^1$.

Problem 2 (MATLAB)

Determine whether the given sets of vectors are linearly independent or linearly dependent.

§5.4, Exercise 9.(MATLAB)

$$v_1 = (2, 1, 3, 4) \quad v_2 = (-4, 2, 3, 1) \quad v_3 = (2, 9, 21, 22) \tag{1*}$$

Problem 3 (MATLAB)

§5.4, Exercise 12.(MATLAB) Perform the following experiments.

(a) Use MATLAB to choose randomly three column vectors in $\mathbb{R}^3$. The MATLAB commands to choose these vectors are:

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y1 = rand(3,1)
y2 = rand(3,1)
y3 = rand(3,1)
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Use the methods of this section to determine whether these vectors are linearly independent or linearly dependent.

(b) Now perform this exercise five times and record the number of times a linearly independent set of vectors is chosen and the number of times a linearly dependent set is chosen.

(c) Repeat the experiment in (b) — but this time randomly choose four vectors in $\mathbb{R}^3$ to be in your set.

Problem 4

§5.5, Exercise 1. Show that $\mathcal{U} = \{u_1, u_2, u_3\}$ where

$$u_1 = (1, 1, 0) \quad u_2 = (0, 1, 0) \quad u_3 = (-1, 0, 1)$$

is a basis for $\mathbb{R}^3$.

Problem 5

§5.5, Exercise 2. Let $S = \mathrm{span}\{v_1, v_2, v_3\}$ where

$$v_1 = (1, 0, -1, 0) \quad v_2 = (0, 1, 1, 1) \quad v_3 = (5, 4, -1, 4).$$

Find the dimension of S and find a basis for S.

Problem 6

§5.5, Exercise 4. Show that the set V of all 2×2 matrices is a vector space. Show that the dimension of V is four by finding a basis of V with four elements. Show that the space $M(m, n)$ of all $m \times n$ matrices is also a vector space. What is $\dim M(m, n)$?

Problem 7

§5.5, Exercise 5. Show that the set $\mathcal{P}_n$ of all polynomials of degree less than or equal to n is a subspace of $\mathcal{C}^1$. What is $\dim \mathcal{P}_2$? What is $\dim \mathcal{P}_n$?

Problem 8

§5.6, Exercise 5. Let A be a 7×5 matrix with $\mathrm{rank}(A) = r$.

(a) What is the largest value that r can have?

(b) Give a condition equivalent to the system of equations $Ax = b$ having a solution.

(c) What is the dimension of the null space of A?

(d) If there is a solution to $Ax = b$, then how many parameters are needed to describe the set of all solutions?

Problem 9

§5.6, Exercise 6. Let

$$A = \begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 1 & 5 & 7 \\ 3 & 4 & 4 & 11 \end{pmatrix}.$$

(a) Find a basis for the subspace $\mathcal{C} \subset \mathbb{R}^3$ spanned by the columns of A.

(b) Find a basis for the subspace $\mathcal{R} \subset \mathbb{R}^4$ spanned by the rows of A.

(c) What is the relationship between $\dim \mathcal{C}$ and $\dim \mathcal{R}$?

Problem 10

§5.6, Exercise 14. Let $\{v_1, v_2, v_3\}$ and $\{w_1, w_2\}$ be linearly independent sets of vectors in a vector space V. Show that if

$$\operatorname{span}\{v_1, v_2, v_3\} \cap \operatorname{span}\{w_1, w_2\} = \{0\}$$

then

$$\dim(\operatorname{span}\{v_1, v_2, v_3, w_1, w_2\}) = 5$$

Hint: First show that if $v \in \operatorname{span}\{v_1, v_2, v_3\}$, $w \in \operatorname{span}\{w_1, w_2\}$, and $v + w = 0$, then $v = w = 0$.

Problem 11

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 15. Every set of three vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$.

Problem 12

In Exercises 15-20 decide whether the statement is true or false, and explain your answer.

§5.6, Exercise 20. If U is a subspace of $\mathbb{R}^3$ of dimension 1 and V is a subspace of $\mathbb{R}^3$ of dimension 2, then $U \cap V = \{0\}$.