Math 2568 Homework 8

Math 2568 Due: Monday, October 21, 2019

Problem 1

Determine whether or not each of the given functions $x_1(t)$ and $x_2(t)$ is a solution to the given differential equation.

§4.1, Exercise 2. ODE: $\frac{dx}{dt} = x + e^t$.

Functions: $x_1(t) = te^t$ and $x_2(t) = 2e^t$.

Problem 2

§4.1, Exercise 6. Solve the differential equation

$$\frac{dx}{dt} = -3x.$$

At what time t_1 will $x(t_1)$ be half of $x(0)$?

Problem 3

Consider the uncoupled system of differential equations (4.3.2). For each choice of a and d, determine whether the origin is a saddle, source, or sink.

§4.3, Exercise 3. $a = 1$ and $d = -1$.

Problem 4

§4.3, Exercise 6. Let $(x(t), y(t))$ be the solution (4.3.3) of (4.3.2) with initial condition $(x(0), y(0)) = (x_0, y_0)$, where $x_0 \neq 0 \neq y_0$.

(a) Show that the points $(x(t), y(t))$ lie on the curve whose equation is:

$$y_0^a x^d - x_0^d y^a = 0.$$

(b) Verify that if $a = 1$ and $d = 2$, then the solution lies on a parabola tangent to the x-axis.

Problem 5 (MATLAB)

§4.3, Exercise 10.(MATLAB) Suppose that $a = d < 0$. Verify experimentally using pplane9 that all trajectories approach the origin along straight lines. Try to prove this conjecture?

Problem 6 (MATLAB)

§4.4, Exercise 1.(MATLAB) Choose the linear system in pplane9 and set $a = 0$, $b = 1$, and $c = -1$. Then find values d such that except for the origin itself all solutions appear to

(a) spiral into the origin;

(b) spiral away from the origin;

(c) form circles around the origin;

Problem 7

Determine which of the function pairs $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are solutions to the given system of ordinary differential equations.

§4.4, Exercise 6. The ODE is:

$$\begin{aligned} \dot{x} &= 2x - 3y \\ \dot{y} &= x - 2y. \end{aligned}$$

The pairs of functions are:

$$(x_1(t), y_1(t)) = e^t(3, 1) \quad \text{and} \quad (x_2(t), y_2(t)) = (e^{-t}, e^{-t}).$$

Problem 8

§4.5, Exercise 2. Show that all solutions to the system of linear differential equations

$$\begin{aligned} \frac{dx}{dt} &= 3x \\ \frac{dy}{dt} &= -2y \end{aligned}$$

are linear combinations of the two solutions

$$U(t) = e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad V(t) = e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Problem 9

§4.5, Exercise 3. Consider

$$\frac{dX}{dt}(t) = CX(t) \tag{1}$$

where

$$C = \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}.$$

Let

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

and let

$$Y(t) = e^{2t} v_1 \quad \text{and} \quad Z(t) = e^{-t} v_2.$$

(a) Show that $Y(t)$ and $Z(t)$ are solutions to (1).

(b) Show that $X(t) = 2Y(t) - 14Z(t)$ is a solution to (1).

(c) Use the principle of superposition to verify that $X(t) = \alpha Y(t) + \beta Z(t)$ is a solution to (1).

(d) Using the general solution found in part (c), find a solution $X(t)$ to (1) such that

$$X(0) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Problem 10

§4.5, Exercise 5. Let

$$C = \begin{pmatrix} a & b \\ b & a \end{pmatrix}.$$

Show that

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

are eigenvectors of C. What are the corresponding eigenvalues?

Problem 11

§4.5, Exercise 6. Let

$$C = \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}.$$

Show that C has no real eigenvectors.

Problem 12

§4.6, Exercise 1. For which values of λ is the matrix

$$\begin{pmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{pmatrix}$$

not invertible? **Note:** These values of λ are just the eigenvalues of the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.