Math 2568 Homework 9
Math 2568 Due: Monday, October 28, 2019

Problem 1

Consider the system of differential equations

$$\begin{array}{rcl} \frac{dx}{dt} & = & 65x + 42y \\ \frac{dy}{dt} & = & -99x - 64y. \end{array} \tag{1}$$

§6.1, Exercise 1. Verify that

$$v_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} -7 \\ 11 \end{pmatrix}$$

are eigenvectors of the coefficient matrix of (2) and find the associated eigenvalues.

Problem 2

Consider the system of differential equations

$$\begin{array}{rcl} \frac{dx}{dt} & = & 65x + 42y \\ \frac{dy}{dt} & = & -99x - 64y. \end{array} \tag{2}$$

§6.1, Exercise 2. Find the solution to (2) satisfying initial conditions $X(0) = (-14, 22)^t$.

Problem 3

Consider the system of differential equations

$$\begin{array}{rcl} \frac{dx}{dt} & = & x - y \\ \frac{dy}{dt} & = & -x + y. \end{array} \tag{3}$$

§6.1, Exercise 5. The eigenvalues of the coefficient matrix of (4) are 0 and 2. Find the associated eigenvectors.

Problem 4

Consider the system of differential equations

$$\begin{array}{rcl} \frac{dx}{dt} & = & x - y \\ \frac{dy}{dt} & = & -x + y. \end{array} \tag{4}$$

§6.1, Exercise 6. Find the solution to (4) satisfying initial conditions $X(0) = (2, -2)^t$.

Problem 5

Consider the system of differential equations

$$\begin{array}{rcl} \frac{dx}{dt} & = & -2x + 7y \\ \frac{dy}{dt} & = & 5y, \end{array} \tag{5}$$

§6.1, Exercise 13. Find a solution to (5) satisfying the initial condition $(x(0), y(0)) = (1, 0)$.

Problem 6

In modern language De Moivre's formula states that

$$e^{ni\theta} = \left(e^{i\theta}\right)^n.$$

In Exercises 2 - 3 use De Moivre's formula coupled with Euler's formula (6.2.5) to determine trigonometric identities for the given quantity in terms of $\cos\theta$, $\sin\theta$, $\cos\varphi$, $\sin\varphi$.

§6.2, Exercise 2. $\cos(\theta + \varphi)$.

Problem 7

Compute the general solution for the given system of differential equations.

§6.2, Exercise 4. $\dfrac{dX}{dt} = \begin{pmatrix} -1 & -4 \\ 2 & 3 \end{pmatrix} X.$

Problem 8

Compute the general solution for the given system of differential equations.

§6.2, Exercise 5. $\dfrac{dX}{dt} = \begin{pmatrix} 8 & -15 \\ 3 & -4 \end{pmatrix} X.$

Problem 9

Compute the general solution for the given system of differential equations.

§6.2, Exercise 6. $\dfrac{dX}{dt} = \begin{pmatrix} 5 & -1 \\ 1 & 3 \end{pmatrix} X.$

Problem 10

Compute the general solution for the given system of differential equations.

§6.2, Exercise 7. $\dfrac{dX}{dt} = \begin{pmatrix} -4 & 4 \\ -1 & 0 \end{pmatrix} X.$