

Problem 1. Rows or columns? Fill in the blank below to make the statement correct.

- (1) Suppose A is an $m \times n$ matrix in reduced row echelon form. The columns of A span \mathbb{R}^m if and only if A has a pivot in every _____.
- (2) Suppose A is an $m \times n$ matrix in reduced row echelon form. The columns of A are linearly independent if and only if A has a pivot in every _____.
- (3) Suppose A is an $m \times n$ matrix. The rank of A is equal to the dimension of the _____ space of A .
- (4) Suppose A is an $m \times n$ matrix. The rank of A plus the nullity of A is equal to the number of _____ of A .
- (5) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$. The equation $A\vec{x} = \vec{b}$ has a solution if and only if \vec{b} belongs to the _____ space of A .

Problem 2. Consider the set of vectors

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

- (1) Find a subset $L \subset S$ which is linearly independent.
- (2) Extend L to a basis for \mathbb{R}^4 .

Problem 3. Consider the matrix

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

- (1) Find the general solution to $AX(t) = X'(t)$.
- (2) Find the particular solution with initial condition $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Problem 4. Find bases for the null, row, and columns spaces of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 1 & 0 \\ 7 & -2 & 1 & 3 \end{pmatrix}.$$

Problem 5. Let v_1, v_2, v_3 be vectors in \mathbb{R}^5 and A is a 4×5 matrix. Suppose Av_1, Av_2, Av_3 are linearly independent vectors in \mathbb{R}^4 . Show that v_1, v_2, v_3 are linearly independent.