Homework 11
Math 2568 April 10, 2019

Problem 1

compute the determinants of the given matrix.

§7.1, Exercise 2. $B = \begin{pmatrix} 1 & 0 & 2 & 3 \\ -1 & -2 & 3 & 2 \\ 4 & -2 & 0 & 3 \\ 1 & 2 & 0 & -3 \end{pmatrix}.$

Problem 2

compute the determinants of the given matrix.

§7.1, Exercise 3. $C = \begin{pmatrix} 2 & 1 & -1 & 0 & 0 \\ 1 & -2 & 3 & 0 & 0 \\ -3 & 2 & -2 & 0 & 0 \\ 1 & 1 & -1 & 2 & 4 \\ 0 & 2 & 3 & -1 & -3 \end{pmatrix}.$

Problem 3

§7.1, Exercise 4. Find $\det(A^{-1})$ where $A = \begin{pmatrix} -2 & -3 & 2 \\ 4 & 1 & 3 \\ -1 & 1 & 1 \end{pmatrix}.$

Problem 4

§7.1, Exercise 15. Suppose that two $n \times p$ matrices A and B are row equivalent. Show that there is an invertible $n \times n$ matrix P such that $B = PA$.

Problem 5

§7.2, Exercise 4. Consider the matrix

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(a) Verify that the characteristic polynomial of A is $p_\lambda(A) = (\lambda - 1)(\lambda + 2)^2$.

(b) Show that $(1, 1, 1)$ is an eigenvector of A corresponding to $\lambda = 1$.

(c) Show that $(1, 1, 1)$ is orthogonal to every eigenvector of A corresponding to the eigenvalue $\lambda = -2$.

Problem 6

§7.2, Exercise 7. Let A be an $n \times n$ matrix. Suppose that

$$A^2 + A + I_n = 0.$$

Prove that A is invertible.

Problem 7

Decide whether the given statements are *true* or *false*. If the statements are false, give a counterexample; if the statements are true, give a proof.

§7.2, Exercise 8. If the eigenvalues of a 2×2 matrix are equal to 1, then the four entries of that matrix are each less than 500.

Problem 8

Decide whether the given statements are *true* or *false*. If the statements are false, give a counterexample; if the statements are true, give a proof.

§7.2, Exercise 9. If A is a 4×4 matrix and $\det(A) > 0$, then $\det(-A) > 0$.

Problem 9

Decide whether the given statements are *true* or *false*. If the statements are false, give a counterexample; if the statements are true, give a proof.

§7.2, Exercise 10. The trace of the product of two $n \times n$ matrices is the product of the traces.

Problem 10

§10.1, Exercise 6. Let A be an $n \times n$ real diagonalizable matrix. Show that $A + \alpha I_n$ is also real diagonalizable.

Problem 11

§10.1, Exercise 9. Let A be an $n \times n$ matrix all of whose eigenvalues equal ± 1. Show that if A is diagonalizable, the $A^2 = I_n$.