Homework 12
Math 2568

Problem 1

§10.1, Exercise 3. Let

$$A = \begin{pmatrix} -1 & 4 & -2 \\ 0 & 3 & -2 \\ 0 & 4 & -3 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of A, and find an invertible matrix S so that $S^{-1}AS$ is diagonal.

Problem 2

§10.1, Exercise 7. Let A be an $n \times n$ matrix with a real eigenvalue λ and associated eigenvector v. Assume that all other eigenvalues of A are different from λ. Let B be an $n \times n$ matrix that commutes with A; that is, $AB = BA$. Show that v is also an eigenvector for B.

Problem 3

Determine the eigenvalues and their geometric and algebraic multiplicities for the given matrix.

§10.3, Exercise 3. $C = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$

Problem 4

Find a basis consisting of the eigenvectors for the given matrix supplemented by generalized eigenvectors. Choose the generalized eigenvectors with lowest index possible.

§10.3, Exercise 5. $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}.$

Problem 5

§8.1, Exercise 1. Use Theorem 8.1.2 and (8.1.3) to construct matrix of a linear mapping L from $\mathbb{R}^3$ to $\mathbb{R}^2$ with $L(v_i) = w_i$, $i = 1, 2, 3$, where

$$v_1 = (1, 0, 2) \quad v_2 = (2, -1, 1) \quad v_3 = (-2, 1, 0)$$

and

$$w_1 = (-1, 0) \quad w_2 = (0, 1) \quad w_3 = (3, 1).$$

Problem 6

§8.1, Exercise 10. Show that

$$\frac{d^2}{dt^2} : \mathcal{P}_4 \to \mathcal{P}_2$$

is a linear mapping. Then compute bases for the null space and range of $\frac{d^2}{dt^2}$.

Problem 7

§8.2, Exercise 3. Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 2 \\ 1 & 2 & -1 & 3 \end{pmatrix}.$$

(a) Find a basis for the row space of A and the row rank of A.

(b) Find a basis for the column space of A and the column rank of A.

(c) Find a basis for the null space of A and the nullity of A.

(d) Find a basis for the null space of A^t and the nullity of A^t.

Problem 8

§8.2, Exercise 5. Let B be an $m \times p$ matrix and let C be a $p \times n$ matrix. Prove that the rank of the $m \times n$ matrix $A = BC$ satisfies

$$\text{rank}(A) \leq \min\{\text{rank}(B),\ \text{rank}(C)\}.$$

Problem 9

§8.3, Exercise 2. Let $w_1 = (1, 2)$ and $w_2 = (0, 1)$ be a basis for $\mathbb{R}^2$. Let $L_A : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by the matrix

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

in standard coordinates. Find the matrix $[L]_{\mathcal{W}}$.

Problem 10 (MATLAB)

§8.3, Exercise 5.(MATLAB) Let

$$w_1 = (1, 0, 2), \quad w_2 = (2, 1, 4), \quad \text{and} \quad w_3 = (0, 1, -1)$$

be a basis for $\mathbb{R}^3$. Find $[v]_{\mathcal{W}}$ where $v = (2, 1, 5)$.

Problem 11 (MATLAB)

§8.3, Exercise 8.(MATLAB) Let A be the 4×4 matrix

$$A = \begin{pmatrix} 2 & 1 & 4 & 6 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 2 & 1 & 1 & 5 \end{pmatrix} \tag{1}$$

and let $\mathcal{W} = \{w_1, w_2, w_3, w_4\}$ where

$$\begin{aligned} w_1 &= (1, 2, 3, 4) \\ w_2 &= (0, -1, 1, 3) \\ w_3 &= (2, 0, 0, 1) \\ w_4 &= (-1, 1, 3, 0) \end{aligned} \tag{2}$$

Verify that $\mathcal{W}$ is a basis of $\mathbb{R}^4$ and compute the matrix associated to A in the $\mathcal{W}$ basis.