Homework 3

Math 2568 Due: January 30, 2019

Problem 1

§3.1, Exercise 7. Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

Denote the columns of the matrix A by

$$A_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}, \quad A_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}, \quad \cdots \quad A_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}.$$

Show that the matrix vector product Ax can be written as

$$Ax = x_1 A_1 + x_2 A_2 + \cdots + x_n A_n,$$

where $x_j A_j$ denotes scalar multiplication (see Chapter 1).

Problem 2

§3.1, Exercise 8. Let

$$C = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Find a 2-vector z such that $Cz = b$.

Problem 3 (MATLAB)

§3.1, Exercise 16.(MATLAB) Let

$$A = \begin{pmatrix} 2 & 4 & -1 \\ 1 & 3 & 2 \\ -1 & -2 & 5 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}. \tag{1}$$

Find a 3-vector x such that $Ax = b$.

Problem 4

Find a nonzero vector that is mapped to the origin by the given matrix.

§3.2, Exercise 1. $A = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix}$.

Problem 5

§3.2, Exercise 5. What 2×2 matrix rotates the plane clockwise by $45°$?

Problem 6 (MATLAB)

Use `map` to help describe the planar motions of the associated linear mappings for the given 2×2 matrix.

§3.2, Exercise 21.(MATLAB) $A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$.

Problem 7

§3.3, Exercise 11. The *cross product* of two 3-vectors $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ is the 3-vector

$$x \times y = (x_2 y_3 - x_3 y_2, -(x_1 y_3 - x_3 y_1), x_1 y_2 - x_2 y_1).$$

Let $K = (2, 1, -1)$. Show that the mapping $L : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$L(x) = x \times K$$

is a linear mapping. Find the 3×3 matrix A such that

$$L(x) = Ax,$$

that is, $L = L_A$.

Problem 8

Determine whether the given transformation is linear.

§3.3, Exercise 8. $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2 - 1)$.

Problem 9 (MATLAB)

Use MATLAB to verify (3.3.1) and (3.3.2).

§3.3, Exercise 18.(MATLAB)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 4 & 0 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}, \quad y = \begin{pmatrix} 0 \\ -5 \\ 10 \end{pmatrix}, \quad c = 21; \qquad (2)$$