Homework 5
Math 2568

Problem 1

§3.3, Exercise 13. Let σ permute coordinates cyclically in $\mathbb{R}^3$; that is,

$$\sigma(x_1, x_2, x_3) = (x_2, x_3, x_1).$$

Find a 3×3 matrix A such that $\sigma = L_A$.

Problem 2

§3.7, Exercise 13. Let A and B be 3×3 invertible matrices so that

$$A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \quad \text{and} \quad B^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Without computing A or B, determine the following:

(a) $\operatorname{rank}(A)$

(b) The solution to

$$Bx = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(c) $(2BA)^{-1}$

(d) The matrix C so that $ACB + 3I_3 = 0$.

Problem 3

§3.8, Exercise 3. Show that the 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is row equivalent to I_2 if and only if $ad - bc \neq 0$. **Hint:** Prove this result separately in the two cases $a \neq 0$ and $a = 0$.

Problem 4

§3.8, Exercise 4. Let A be a 2×2 matrix having integer entries. Find a condition on the entries of A that guarantees that A^{-1} has integer entries.

Problem 5

Use Cramer's rule (3.8.5) to solve the given system of linear equations.

§3.8, Exercise 8. Solve
$$\begin{array}{rcl} 2x + 3y & = & 2 \\ 3x - 5y & = & 1 \end{array}$$
for x.

Problem 6 (MATLAB)

§3.8, Exercise 10.(MATLAB) Use MATLAB to choose five 2×2 matrices at random and compute their inverses. Do you get the impression that 'typically' 2×2 matrices are invertible? Try to find a reason for this fact using the determinant of 2×2 matrices.

Problem 7

Determine whether or not each of the given functions $x_1(t)$ and $x_2(t)$ is a solution to the given differential equation.

§4.1, Exercise 1. ODE: $\frac{dx}{dt} = \frac{t}{x-1}$.

Functions: $x_1(t) = t + 1$ and $x_2(t) = \frac{1 + \sqrt{4t^2 + 1}}{2}$.

Problem 8

§4.1, Exercise 6. Solve the differential equation

$$\frac{dx}{dt} = -3x.$$

At what time t_1 will $x(t_1)$ be half of $x(0)$?

Problem 9

Consider the uncoupled system of differential equations (4.3.2). For each choice of a and d, determine whether the origin is a saddle, source, or sink.

§4.3, Exercise 4. $a = -0.01$ and $d = -2.4$.

Problem 10

§4.3, Exercise 6. Let $(x(t), y(t))$ be the solution (4.3.3) of (4.3.2) with initial condition $(x(0), y(0)) = (x_0, y_0)$, where $x_0 \neq 0 \neq y_0$.

(a) Show that the points $(x(t), y(t))$ lie on the curve whose equation is:

$$y_0^a x^d - x_0^d y^a = 0.$$

(b) Verify that if $a = 1$ and $d = 2$, then the solution lies on a parabola tangent to the x-axis.