Homework 6
Math 2568

Problem 1 (MATLAB)

§4.4, Exercise 3.(MATLAB) Choose the linear system in pplane9 and set $a = d$ and $b = c$. Verify that for these systems of differential equations:

(a) When $|a| < b$ typical trajectories approach the line $y = x$ as $t \to \infty$ and the line $y = -x$ as $t \to -\infty$.

(b) Assume that b is positive, a is negative, and $b < -a$. With these assumptions show that the origin is a sink and that typical trajectories approach the origin tangent to the line $y = x$.

Problem 2

Determine which of the function pairs $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are solutions to the given system of ordinary differential equations.

§4.4, Exercise 6. The ODE is:

$$\begin{aligned} \dot{x} &= 2x - 3y \\ \dot{y} &= x - 2y. \end{aligned}$$

The pairs of functions are:

$$(x_1(t), y_1(t)) = e^t(3, 1) \quad \text{and} \quad (x_2(t), y_2(t)) = (e^{-t}, e^{-t}).$$

Problem 3

§4.5, Exercise 4. Find a solution to

$$\dot{X}(t) = CX(t)$$

where

$$C = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

and

$$X(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Hint: Observe that
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
are eigenvectors of C.

Problem 4 (MATLAB)

§4.5, Exercise 11.(MATLAB) Use MATLAB to verify that solutions to the system of linear differential equations

$$\begin{aligned} \frac{dx}{dt} &= 2x + y \\ \frac{dy}{dt} &= y \end{aligned}$$

are linear combinations of the two solutions

$$U(t) = e^{2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad V(t) = e^{t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

More concretely, proceed as follows:

(a) By superposition, the general solution to the differential equation has the form $X(t) = \alpha U(t) + \beta V(t)$. Find constants α and β such that $\alpha U(0) + \beta V(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(b) Graph the second component $y(t)$ of this solution using the MATLAB `plot` command.

(c) Use pplane9 to compute a solution via the Keyboard input starting at $(x(0), y(0)) = (0, 1)$ and then use the `y vs t` command in pplane9 to graph this solution.

(d) Compare the results of the two plots.

(e) Repeat steps (a)–(d) using the initial vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Problem 5

§4.5, Exercise 6. Let
$$C = \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}.$$

Show that C has no real eigenvectors.

Problem 6

§4.6, Exercise 1. For which values of λ is the matrix

$$\begin{pmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{pmatrix}$$

not invertible? **Note:** These values of λ are just the eigenvalues of the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$.

Problem 7

Compute the determinant, trace, and characteristic polynomials for the given 2×2 matrix.

§4.6, Exercise 2. $\begin{pmatrix} 1 & 4 \\ 0 & -1 \end{pmatrix}$.

Problem 8

Compute the eigenvalues for the given 2×2 matrix.

§4.6, Exercise 6. $\begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix}$.

Problem 9

Compute the eigenvalues for the given 2×2 matrix.

§4.6, Exercise 8. $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$.

Problem 10 (MATLAB)

§4.6, Exercise 15. (MATLAB) The MATLAB command `eig` computes the eigenvalues of matrices. Use `eig` to compute the eigenvalues of $A = \begin{pmatrix} 2.34 & -1.43 \\ \pi & e \end{pmatrix}$.