Homework 8
Math 2568

Problem 1

A single equation in three variables is given. For each equation write the subspace of solutions in $\mathbb{R}^3$ as the span of two vectors in $\mathbb{R}^3$.

§5.2, Exercise 1. $4x - 2y + z = 0$.

Problem 2

§5.2, Exercise 9. Write a system of two linear equations of the form $Ax = 0$ where A is a 2×4 matrix whose subspace of solutions in $\mathbb{R}^4$ is the span of the two vectors

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

Problem 3

§5.2, Exercise 20. Let $Ax = b$ be a system of m linear equations in n unknowns, and let $r = \operatorname{rank}(A)$ and $s = \operatorname{rank}(A|b)$. Suppose that this system has a unique solution. What can you say about the relative magnitudes of m, n, r, s?

Problem 4 (MATLAB)

§5.3, Exercise 5.(MATLAB) Use row reduction to find the solutions to $Ax = 0$ where A is given in (1). Does your answer agree with the MATLAB answer using `null`? If not, explain why.

$$A = \begin{pmatrix} -4 & 0 & -4 & 3 \\ -4 & 1 & -1 & 1 \end{pmatrix} \tag{1}$$

Problem 5

§5.4, Exercise 1. Let w be a vector in the vector space V. Show that the sets of vectors $\{w, 0\}$ and $\{w, -w\}$ are linearly dependent.

Problem 6

§5.4, Exercise 3. Let

$$u_1 = (1, -1, 1) \quad u_2 = (2, 1, -2) \quad u_3 = (10, 2, -6).$$

Is the set $\{u_1, u_2, u_3\}$ linearly dependent or linearly independent?

Problem 7

§5.4, Exercise 7. Suppose that the three vectors $u_1, u_2, u_3 \in \mathbb{R}^n$ are linearly independent. Show that the set

$$\{u_1 + u_2, u_2 + u_3, u_3 + u_1\}$$

is also linearly independent.

Problem 8 (MATLAB)

§5.4, Exercise 11.(MATLAB) Perform the following experiments.

(a) Use MATLAB to choose randomly three column vectors in $\mathbb{R}^3$. The MATLAB commands to choose these vectors are:

```
y1 = rand(3,1)
y2 = rand(3,1)
y3 = rand(3,1)
```

Use the methods of this section to determine whether these vectors are linearly independent or linearly dependent.

(b) Now perform this exercise five times and record the number of times a linearly independent set of vectors is chosen and the number of times a linearly dependent set is chosen.

(c) Repeat the experiment in (b) — but this time randomly choose four vectors in $\mathbb{R}^3$ to be in your set.

Problem 9

§5.5, Exercise 1. Show that $\mathcal{U} = \{u_1, u_2, u_3\}$ where

$$u_1 = (1, 1, 0) \quad u_2 = (0, 1, 0) \quad u_3 = (-1, 0, 1)$$

is a basis for $\mathbb{R}^3$.

Problem 10

§5.5, Exercise 2. Let $S = \text{span}\{v_1, v_2, v_3\}$ where

$$v_1 = (1, 0, -1, 0) \quad v_2 = (0, 1, 1, 1) \quad v_3 = (5, 4, -1, 4).$$

Find the dimension of S and find a basis for S.