Problem 1. Rows or columns? Fill in the blank below to make the statement correct.

(1) Suppose A is an $m \times n$ matrix in reduced row echelon form. The columns of A span $\mathbb{R}^m$ if and only if A has a pivot in every __________.
(2) Suppose A is an $m \times n$ matrix in reduced row echelon form. The columns of A are linearly independent if and only if A has a pivot in every __________.
(3) Suppose A is an $m \times n$ matrix. The rank of A is equal to the dimension of the __________ space of A.
(4) Suppose A is an $m \times n$ matrix. The rank of A plus the nullity of A is equal to the number of __________ of A.
(5) To see if the vectors $\{v_1, \ldots, v_n\}$ are linearly independent, we form a matrix A by letting the v_i be the __________ of A. We then row reduce and see if there are n pivots.

Problem 2. Consider the set of vectors

$$S = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

(1) Find a subset $L \subset S$ which is linearly independent.
(2) Extend L to a basis for $\mathbb{R}^4$.

Problem 3. Consider the matrix

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

(1) Find the general solution to $AX(t) = X'(t)$.
(2) Find the particular solution with initial condition $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Problem 4. Find bases for the null, row, and columns spaces of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 1 & 0 \\ 7 & -2 & 1 & 3 \end{pmatrix}.$$

Problem 5. Let v_1, v_2, v_3 be vectors in $\mathbb{R}^5$ and A is a 4×5 matrix. Suppose Av_1, Av_2, Av_3 are linearly independent vectors in $\mathbb{R}^4$. Show that v_1, v_2, v_3 are linearly independent.