Math 2568 Practice Midterm 2 **March 22, 2019**

Problem 1. True or false? Explain your reasoning.

(1) The set of skew-symmetric 2×2 matrices with entries in $\mathbb{R}$ is a vector space of dimension 2. (A matrix is skew-symmetric if $A^T = -A$.)

(2) Let A be an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ a vector. The set

$$\{\vec{x} \in \mathbb{R}^n : A\vec{x} = \vec{b}\}$$

is always a vector space.

(3) Let V be a vector space of dimension n, and S a subset of vectors in V that spans V. Then S must contain at least n elements.

(4) Let A be a 2×2 matrix whose eigenvalues are 3 and -1. Then A is invertible.

(5) Every invertible matrix is similar to a diagonal matrix.

(6) If W is a subspace of V, then $\dim W < \dim V$.

Problem 2. Let V be a vector space, and let $\{v_1, v_2, v_3\}$ be a linearly independent set of vectors in V. Let $w \in V$ be a vector such that $w \notin \operatorname{span}\{v_1, v_2, v_3\}$. Show that $\{v_1, v_2, v_3, w\}$ is also a linearly independent set.

Problem 3. (1) Let $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Find the Jordan normal form of A, and the matrix P such that the Jordan normal form J of A is equal to $J = P^{-1}AP$.

(2) Given that the matrix B satisfies $B = Q^{-1}JQ$ for the same Jordan normal form as part (a) and for some invertible matrix Q, Show that A is similar to B by explicitly writing down the matrix R (in terms of P, Q) such that

$$B = R^{-1}AR.$$

Problem 4. Find the unique solution to the initial value problem

$$\vec{X}'(t) = \begin{pmatrix} -7 & 9 \\ -4 & 5 \end{pmatrix} \vec{X}(t),$$

with

$$\vec{X}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Problem 5. (1) Give an example of two subspaces U and V of $\mathbb{R}^3$, such that $\dim U = 2$, $\dim V = 1$ and $U \cap V = \{0\}$;

(2) Give an example of two subspaces U and V of $\mathbb{R}^3$, such that $U \cap V = \operatorname{span}\{\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}\}$, and $\dim U = \dim V = 2$.

(3) Give an example of two subspaces U and V of $\mathbb{R}^4$, such that $U = \operatorname{span}\{v_1, v_2\}$, $V = \operatorname{span}\{v_3, v_4\}$, $\mathbb{R}^4 = \operatorname{span}\{v_1, \ldots, v_4\}$.