

MATH 3345 HOMEWORK 9

Let $f : A \rightarrow B$. We'll again use some alternate notation for image and preimage of f . For $X \subseteq A$, we define the *image of X under f* as the set

$$\vec{f}(X) = \{f(x) | x \in X\} \subseteq B.$$

For $Y \subseteq B$, we define the *preimage of Y under f* as the set

$$\overleftarrow{f}(Y) = \{a \in A | f(a) \in Y\}.$$

Recall that the *power set* of A is the set $P(A) = \{X | X \subseteq A\}$. Define $\vec{f} : P(A) \rightarrow P(B)$ by $\vec{f}(X)$ is the image of X under f . Define $\overleftarrow{f} : P(B) \rightarrow P(A)$ by $\overleftarrow{f}(Y)$ is the preimage of Y under f .

Problem 1. Let $f : A \rightarrow B$. Determine a necessary and sufficient condition for

- (a) $\vec{f} : P(A) \rightarrow P(B)$ to be injective,
- (b) $\vec{f} : P(A) \rightarrow P(B)$ to be surjective,
- (c) $\overleftarrow{f} : P(B) \rightarrow P(A)$ to be injective, and
- (d) $\overleftarrow{f} : P(B) \rightarrow P(A)$ to be surjective.

Hint: There are two ways to approach this problem. One way is you could look at Homework 8 Problems 1 and 2. The second way is you could look at Homework 9 Problems 4 and 5, together with the criteria for left and right invertibility.

Problem 2.

- (a) Falkner Section 12 Exercise 16
- (b) Falkner Section 12 Exercise 17

Problem 3. Falkner Section 12 Exercise 18

Problem 4. Falkner Section 13 Exercise 6

Problem 5. Falkner Section 13 Exercise 7