MATH 3345 HOMEWORK 11

Problem 1. Suppose A and B are finite. Find necessary and sufficient conditions for there to be no surjections from A to B.

Problem 2. Suppose A and B are finite. Compute $\#\operatorname{Bij}(A, B)$, the number of bijections from A to B, in terms of $\#A$ and $\#B$.
Hint: You can look at Falkner Section 14 Exercise 11.

Problem 3. Falkner Section 13 Exercise 10

Problem 4. Prove that A is countable and nonempty if and only if there is a surjection $\mathbb{N} \to A$.
Hint: Given a surjection $f : \mathbb{N} \to A$, we can inductively construct a bijection $g : \mathbb{N} \to A$ as follows. First, set $g(1) = f(1)$. Suppose $n \in \mathbb{N}$, and that we have defined $g(1), \ldots, g(n)$. Let $k_n \in \mathbb{N}$ be the smallest $k \in \mathbb{N}$ such that $f(k) \notin \{g(1), \ldots, g(n)\}$. Define $g(n+1) = f(k_n)$.

(a) Why does k_n exist?

(b) Show that g is injective.

(c) Show that (k_n) is an increasing sequence, i.e., $k_n < k_{n+1}$.

(d) Show that $f(\{1, \ldots, k_n - 1\}) = g(\{1, \ldots, n\})$ and $f(\{1, \ldots, k_n\}) = g(\{1, \ldots, n, n+1\})$.

(e) Show that g is surjective.

Problem 5. Falkner Section 15 Exercise 1

Optional problem 1. Find a bijection between $\operatorname{Surj}(A, B)$, the set of surjections from A to B, and the set of **ordered** partitions of A with $\#B$ elements. That means $\#B$-tuples $(S_1, \ldots, S_{\#B})$ of nonempty, pairwise disjoint subsets of A whose union is A. Why do you need ordered partitions?