

MATH 3345 HOMEWORK 4

Problem 1. Section 4 Exercise 14 from Falkner

Problem 2.

(a) Section 4 Exercise 22 from Falkner

Hint: Try proving the contrapositive. That is, suppose $p \in \mathbb{N}$ and $p \geq 2$. Write down what it means for p to be not prime.

(b) Explain why the statement in Exercise 22 is the converse of the statement in Remark 4.54 right before Exercise 22. (Note: This is Remark 4.52 in the old version of Falkner's notes.)

Problem 3. Suppose that $m \in \mathbb{N}$ with $m \geq 2$. Prove that m is not prime if and only if there is an integer $2 \leq k \leq \sqrt{m}$ such that k divides m .

Hint: Write down what it means for m to be not prime.

Problem 4. Section 4 Exercise 25 from Falkner.

Problem 5. Prove that the following two statements are equivalent for $p \in \mathbb{N}$ with $p \geq 2$.

(a) p is prime.

(b) For all $a, b \in \mathbb{Z}$, $ab \equiv 0 \pmod{p}$ implies $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$.