MATH 3345 HOMEWORK 8

Let $f : A \to B$. For $X \subseteq A$, we define the *image of* X *under* f as the set

$$f(X) = \{f(x) | x \in X\} \subseteq B.$$

For $Y \subseteq B$, we define the *preimage of* Y *under* f as the set

$$f^{-1}(Y) = \{a \in A | f(a) \in Y\}.$$

Problem 1. Let $f : A \to B$.

(a) Show that if $X \subseteq A$, then $X \subseteq f^{-1}(f(X))$.

(b) Find an example where $X \neq f^{-1}(f(X))$.
Hint: try using finite sets.

(c) Find a condition on f which implies $X = f^{-1}(f(X))$.

Problem 2. Let $f : A \to B$.

(a) Show that if $Y \subseteq B$, then $f(f^{-1}(Y)) \subseteq Y$.

(b) Find an example where $f(f^{-1}(Y)) \neq Y$.
Hint: try using finite sets.

(c) Find a condition on f which implies $f(f^{-1}(Y)) = Y$.

Problem 3. Falkner Section 17 Exercise 6

Problem 4.

(a) Falkner Section 17 Exercise 9

(b) Falkner Section 17 Exercise 14

Problem 5. Falkner Section 17 Exercise 16
Hint: Use the Principle of Strong Mathematical Induction. Argue as follows for the inductive step. Look at the set of all partitions of $\{1, \ldots, n+2\}$. *We can* **partition the set of all partitions** *by the size of the set containing* 1. *For example, if* $n = 1$, *we can partition the 5 partitions of* $\{1, 2, 3\}$ *into:*

$$\left\{\{\{1\},\{2\},\{3\}\},\{\{1\},\{2,3\}\}\right\} \cup \left\{\{\{1,2\},\{3\}\}\{\{1,3\},\{2\}\}\right\} \cup \left\{\{\{1,2,3\}\}\right\},$$

For each $k \in \{0, \ldots, n+1\}$, *how many partitions of* $\{1, \ldots, n+2\}$ *are there for which* 1 *is in a subset of size* $k+1$? *First, you need to choose* k *other elements of* $\{2, \ldots, n+1\}$. *How many ways are there to do this? Then you need to look at the number of ways to partition the remaining* $(n+2) - (k+1) = n - k + 1$ *elements. Use the Inductive Hypotheses here. (You* **don't** *need the recursive relation on* $\binom{n}{k}$.*)*