

MATH 3345 HOMEWORK 9

Problem 1. (Falkner Section 11 Exercise 20)

- (a) Let $g : [0, 1) \rightarrow [0, \infty)$ by $g(x) = x/(1 - x)$. Prove that g is a bijection. Find its inverse function $g^{-1} : [0, \infty) \rightarrow [0, 1)$.
- (b) Let $h : (-1, 0) \rightarrow (-\infty, 0)$ by $h(x) = x/(1 + x)$. Prove that h is a bijection and find its inverse function.

Problem 2. Falkner Section 11 Exercise 22

Problem 3. Falkner Section 11 Exercise 23

Let $f : A \rightarrow B$. For the remainder of the homework, we'll use some alternate notation for image and preimage of f . For $X \subseteq A$, we define the *image of X under f* as the set

$$\vec{f}(X) = \{f(x) | x \in X\} \subseteq B.$$

For $Y \subseteq B$, we define the *preimage of Y under f* as the set

$$\overleftarrow{f}(Y) = \{a \in A | f(a) \in Y\}.$$

Recall that the *power set* of A is the set $P(A) = \{X | X \subseteq A\}$. Define $\vec{f} : P(A) \rightarrow P(B)$ by $\vec{f}(X)$ is the image of X under f . Define $\overleftarrow{f} : P(B) \rightarrow P(A)$ by $\overleftarrow{f}(Y)$ is the preimage of Y under f .

Problem 4. Recall $\text{id}_A : A \rightarrow A$ by $a \mapsto a$ for all $a \in A$. Let $f : A \rightarrow A$.

- (a) Show that $\vec{f} = \text{id}_{P(A)}$ if and only if $f = \text{id}_A$.
- (b) Show that $\overleftarrow{f} = \text{id}_{P(A)}$ if and only if $f = \text{id}_A$.

Problem 5. Let $f : A \rightarrow B$, $g : B \rightarrow C$, and $h = g \circ f$. Prove that

- (a) $\vec{h} = \vec{g} \circ \vec{f}$, and
- (b) $\overleftarrow{h} = \overleftarrow{f} \circ \overleftarrow{g}$.

You must show that these functions have the same domain, codomain, and rule.