Math 3345 Practice Exam 1 Answers

(1) (a) False. The negation is "The Buckeyes sometimes lose."
  (b) False. It's a tautology.
  (c) True
  (d) True
  (e) False. Zero divides zero. (Work out why this is true!)

(2) (a) We did this in class. Look at your notes to find a proof.
  (b) We'll prove the contrapositive, which is the statement "If $x, y \in \mathbb{Z}$ are both even, then $x + y$ is even."

  Proof. Suppose $x, y \in \mathbb{Z}$ are both even. Then there exist $k, \ell \in \mathbb{Z}$ such that $x = 2k$ and $y = 2\ell$. Then

$$x + y = 2k + 2\ell = 2(k + \ell).$$

  Since $k + \ell \in \mathbb{Z}$, we see $x + y$ is even. □

(3) (a) The negation of (a) is just False! It turns out that (a) is a tautology, and the negation of a tautology is False.
  The negation is given by

$$(P \Rightarrow Q) \wedge \neg[(Q \Rightarrow R) \Rightarrow (P \Rightarrow R)]$$

  which expands further to

$$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \wedge \neg(P \Rightarrow R).$$

  But we know that $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$, so the above implies $(P \Rightarrow R) \wedge \neg(P \Rightarrow R)$, which is False.
  (b) The negation is given by

$$(\exists a \in A)(\forall b \in B) P(a) \wedge \neg Q(b).$$

(4) Suppose a divides b and b divides c. Then there are $k, \ell \in \mathbb{Z}$ such that $b = ak$ and $c = b\ell$. Then $c = b\ell = (ak)\ell = a(k\ell)$. But since $k\ell \in \mathbb{Z}$, we see a divides c.

(5) We use the method of conditional proof.

(A1) Assume $P \Rightarrow Q$. We must show that $(P \wedge (Q \Rightarrow R)) \Rightarrow R$.

(A2) Assume $P \wedge (Q \Rightarrow R)$. We must show R.

By (A2), we have P. By (A1), we have $P \Rightarrow Q$. We have P and $P \Rightarrow Q$, so by modus ponens, we have Q.

By (A2) again, we have $Q \Rightarrow R$. We now have Q and $Q \Rightarrow R$, so by modus ponens, we have R.

Thus we see under (A2), we have $(P \wedge (Q \Rightarrow R)) \Rightarrow R$.

Discharging (A2), we see that $(P \wedge (Q \Rightarrow R)) \Rightarrow R$ under (A1) alone.

Discharging (A1), we see $(P \Rightarrow Q) \Rightarrow [(P \wedge (Q \Rightarrow R)) \Rightarrow R]$ under no assumptions.