Math 3345 Exam 3 Name:______________________________

December 2, 2016 SID:______________________________

I have read and understood the Code of Student Conduct, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature:____

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

There are 9 problems worth 10 points each.

Instructor's use only:

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Problem 1. (10 points) Use the method of conditional proof to explain in words why the sentence

$$(P \Rightarrow Q) \Rightarrow \{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\}$$

is a tautology. Be careful not to skip any steps. Be explicit about discharging assumptions.

Problem 2. (10 points) What amounts of postage can be formed only using 5 cent and 6 cent stamps? Formulate a conjecture and prove it.
Hint: You may eventually want to use the principle of mathematical induction.

Problem 3. (10 points) Show that every natural number is a product of primes.

Problem 4. (10 points)

(a) Prove that for any $n \in \mathbb{Z}$, $n^2 \equiv 0 \mod 4$ or $n^2 \equiv 1 \mod 4$.

(b) Suppose $n = 4k + 3$ for some $k \in \mathbb{Z}$ with $k \geq 0$. Prove that n cannot be written as the sum of the squares of two integers.

Problem 5. (10 points) Let $R_1 \subset A \times A$ and $R_2 \subset A \times A$ be two equivalence relations on a set A. Does it automatically follow that $R_1 \cup R_2 \subset A \times A$ is an equivalence relation? Either provide a proof that $R_1 \cup R_2$ is an equivalence relation, or provide a specific counterexample.

Problem 6. (10 points) Suppose $f : A \to B$ is surjective and $g : B \to C$ is surjective. Prove that $g \circ f$ is surjective.

Problem 7. (10 points) Show that $f : A \to B$ is injective if and only if f has a left inverse.

Problem 8. (10 points) Let A be a set.

(a) Suppose A is infinite. Inductively define an injective function $\mathbb{N} \to A$.

(b) Suppose there is an injection $\mathbb{N} \to A$. Prove that A is infinite.
Hint: Prove the contrapositive, i.e., if A is finite, then any map $\mathbb{N} \to A$ is not injective. Try using restriction and the Pigeonhole Principle.

Problem 9. (10 points) Show that $\text{Fun}(\mathbb{N}, \{0, 1\})$, the set of all functions from $\mathbb{N}$ to $\{0, 1\}$, is uncountable.