

September 9, 2016

SID: \_\_\_\_\_

I have read and understood the Code of Student Conduct, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature: \_\_\_\_\_

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

There are 5 problems worth 10 points each.

**Instructor's use only:**

Problem	Points
1	
2	
3	
4	
5	
Total	

**Problem 1.** Below is a list of statements. Decide which are true and which are false. On the left of each, write “TRUE” or “FALSE” in capital letters. If the statement is false, correct the statement so that it is true (without simply negating the statement).

There is no partial credit on this problem.

\_\_\_\_\_ (1) (2 points) The negation of “The Buckeyes always win” is “The Buckeyes always lose.”

\_\_\_\_\_ (2) (2 points) The sentence  $P \wedge (P \Rightarrow Q) \Rightarrow Q$  is a contradiction.

\_\_\_\_\_ (3) (2 points)  $P \Rightarrow Q$  is logically equivalent to  $\neg P \vee Q$ .

\_\_\_\_\_ (4) (2 points) For every real number  $x$ ,  $x^2 + 2x + 2 > 0$ .

\_\_\_\_\_ (5) (2 points) Zero divides no integer.

**Problem 2.** (10 points)

- (a) Prove that the sum of two odd integers is even.
- (b) Suppose  $x, y \in \mathbb{Z}$  and  $x + y$  is odd. Prove that  $x$  is odd or  $y$  is odd.

**Problem 3.** (10 points) Compute the negations of the following sentences. Simplify your answers as much as possible.

(a)  $(P \Rightarrow Q) \Rightarrow [(Q \Rightarrow R) \Rightarrow (P \Rightarrow R)]$

(b)  $(\forall x \in A)(\exists b \in B)(P(a) \Rightarrow Q(b)).$

**Problem 4.** (10 points) Suppose  $a, b, c \in \mathbb{Z}$ . Prove that if  $a$  divides  $b$  and  $b$  divides  $c$ , then  $a$  divides  $c$ .

**Problem 5.** (10 points) Use the method of conditional proof to explain in words why the sentence

$$[(P \Rightarrow Q)] \Rightarrow [(P \wedge (Q \Rightarrow R)) \Rightarrow R]$$

is a tautology. Be careful not to skip any steps. Be explicit about discharging assumptions.