Math 3345 Practice Exam 2 Name:____________________

October 18, 2016 SID:____________________

I have read and understood the Code of Student Conduct, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature:________________________________________

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

There are 5 problems worth 10 points each.

**Instructor's use only:**

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

**Problem 1.** (10 points) Prove there are infinitely many primes.

**Problem 2.** (10 points) Prove using the Principle of Mathematical induction that for every natural number $n \geq 3$,

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6}.$$

You may use without proof the fact that for such $n \geq 3$, $\binom{n}{2} = \frac{n(n-1)}{2}$.

**Problem 3.** (10 points) Fix $m \in \mathbb{N}$. For $a \in \mathbb{Z}$, let

$$\overline{a} = \{k \in \mathbb{Z} | a \equiv k \mod m\}.$$

Prove that for all $a, b \in \mathbb{Z}$, $\overline{a} = \overline{b}$ if and only if $a \equiv b \mod m$. You may use that "congruence modulo $m$" is an equivalence relation on $\mathbb{Z}$, i.e., it is reflexive, symmetric, and transitive. But you may not use any general theorems about equivalence relations.

**Problem 4.** (10 points) Prove that a number $k \in \mathbb{Z}$ is divisible by 9 if and only if the sum of its digits is divisible by 9.
*Hint: write*

$$k = d_n 10^n + d_{n-1} 10^{n-1} + \cdots + d_1 10^1 + d_0,$$

*and think about their 'remainders' modulo* 9.

**Problem 5.** (10 points) Suppose $X$ is a set, and $A, B, C$ are all subsets of $X$. Show that the following three sets are equal:

(a) $A \cap (B \setminus C)$

(b) $(A \cap B) \setminus C$

(c) $(A \setminus C) \cap B$.

Give a proof in words. For each equality, you may give a suitable chain of "iffs."