Problem 1. (Falkner Section 11 Exercise 20)
(a) Let \( g : [0, 1) \to [0, \infty) \) by \( g(x) = x/(1 - x) \). Prove that \( g \) is a bijection. Find its inverse function \( g^{-1} : [0, \infty) \to [0, 1) \).

(b) Let \( h : (-1, 0) \to (-\infty, 0) \) by \( h(x) = x/(1 + x) \). Prove that \( h \) is a bijection and find its inverse function.

Problem 2. Falkner Section 11 Exercise 22

Problem 3. Falkner Section 11 Exercise 23

Let \( f : A \to B \). For the remainder of the homework, we’ll use some alternate notation for image and preimage of \( f \). For \( X \subseteq A \), we define the image of \( X \) under \( f \) as the set
\[
\overrightarrow{f}(X) = \{ f(x) \mid x \in X \} \subseteq B.
\]
For \( Y \subseteq B \), we define the preimage of \( Y \) under \( f \) as the set
\[
\overleftarrow{f}(Y) = \{ a \in A \mid f(a) \in Y \}.
\]
Recall that the power set of \( A \) is the set \( P(A) = \{ X \mid X \subseteq A \} \). Define \( \overrightarrow{f} : P(A) \to P(B) \) by \( \overrightarrow{f}(X) \) is the image of \( X \) under \( f \). Define \( \overleftarrow{f} : P(B) \to P(A) \) by \( \overleftarrow{f}(Y) \) is the preimage of \( Y \) under \( f \).

Problem 4. Recall \( \text{id}_A : A \to A \) by \( a \mapsto a \) for all \( a \in A \). Let \( f : A \to A \).

(a) Show that \( \overrightarrow{f} = \text{id}_{P(A)} \) if and only if \( f = \text{id}_A \).
(b) Show that \( \overleftarrow{f} = \text{id}_{P(A)} \) if and only if \( f = \text{id}_A \).

Problem 5. Let \( f : A \to B \), \( g : B \to C \), and \( h = g \circ f \). Prove that
(a) \( \overrightarrow{h} = \overrightarrow{g} \circ \overrightarrow{f} \), and
(b) \( \overleftarrow{h} = \overleftarrow{f} \circ \overleftarrow{g} \).

You must show that these functions have the same domain, codomain, and rule.